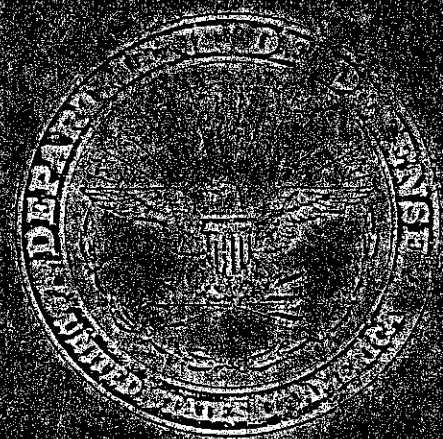


Design of Wood Aircraft Structures



MUNITIONS BOARD
AIRCRAFT COMMITTEE

CONTENTS

CHAPTER 1. GENERAL

	Page
1.0. PURPOSE AND USE OF BULLETIN.....	1
1.00. Introduction.....	1
1.01. Scope of Bulletin.....	1
1.02. Acknowledgments.....	1
1.1. NOMENCLATURE.....	1
1.10. Definitions for Plywood Elements--Beams, Prisms, and Columns in Compression, Strips in Tension..	1
1.11. Definitions for Plywood Panels.....	1
1.12. Standard Structural Symbols for Chapter 2.....	2

CHAPTER 2. STRENGTH OF WOOD AND PLYWOOD ELEMENTS

2.0. PHYSICAL CHARACTERISTICS AND FACTORS AFFECTING THE STRENGTH OF WOOD.....	9
2.00. Anisotropy of Wood.....	9
2.01. Density or Apparent Specific Gravity.....	9
2.02. Moisture Content.....	9
2.03. Shrinkage.....	11
2.04. Temperature.....	12
2.05. Fatigue Properties.....	14
2.06. Plastic Properties.....	14
2.07. Impact Properties.....	14
2.08. Effect of Liquids.....	15
2.09. Miscellaneous Physical Properties.....	15
2.091. Coefficient of Expansion.....	15
2.092. Thermal Conductivity.....	17
2.093. Ignition Temperature.....	18
2.094. Electrical Properties.....	18
2.095. Damping Capacity.....	18
2.1. BASIC STRENGTH AND ELASTIC PROPERTIES OF WOOD.....	20
2.10. Design Values, Tables 2-6 and 2-7.....	20
2.100. Supplemental Notes.....	20
2.1000. Compression Perpendicular to Grain.....	20
2.1001. Compression Parallel to Grain.....	20
2.11. Notes on the Use of Values in Tables 2-6 and 2-7.....	20
2.110. Relation of Design Values in Tables 2-6 and 2-7 to Slope of Grain.....	20
2.111. Tension Parallel to Grain.....	27
2.112. Tension Perpendicular to Grain.....	27
2.12. Standard Test Procedures.....	27
2.120. Static Bending.....	27
2.1200. Modulus of Elasticity (E_L).....	27
2.1201. Fiber Stress at Proportional Limit (F_{bp}).....	27
2.1202. Modulus of Rupture (F_{bu}).....	28
2.1203. Work to Maximum Load.....	28
2.121. Compression Parallel to Grain.....	28
2.1210. Modulus of Elasticity (E_{Lc}).....	28
2.1211. Fiber Stress at Proportional Limit (F_{cp}).....	29
2.1212. Maximum Crushing Strength (F_{cu}).....	29
2.122. Compression Perpendicular to Grain.....	29
2.123. Shear Parallel to Grain (F_{su}).....	29
2.124. Hardness.....	29
2.125. Tension Perpendicular to Grain (F_{tuT}).....	31
2.13. Elastic Properties Not Included in Tables 2-6 and 2-7.....	31

	Page
2.130. Moduli of Elasticity Perpendicular to Grain (E_T , E_R).....	31
2.131. Moduli of Rigidity (G_{LT} , G_{LR} , G_{RT}).....	31
2.132. Poisson's Ratios (μ).....	31
2.14. Stress-strain Relations.....	31
2.15. Strength Under Multiaxial Stress.....	31
2.16. Stress Concentrations.....	32
2.161. Stress Concentrations Around a Hole in a Tension or Compression Member.....	32
2.162. Stress Concentrations Due to a Hole Which is Not Small Compared to the Size of the Member.....	34
2.2. COLUMNS.....	34
2.20. Primary Failure.....	34
2.21. Local Buckling and Twisting Failure.....	34
2.22. Lateral Buckling.....	34
2.3. BEAMS.....	36
2.30. Form Factors.....	36
2.31. Torsional Instability.....	36
2.32. Combined Loadings.....	36
2.320. General.....	36
2.321. Bending and Compression.....	36
2.322. Bending and Tension.....	40
2.33. Shear Webs.....	40
2.34. Beam Section Efficiency.....	40
2.4. TORSION.....	41
2.40. General.....	41
2.41. Torsional Properties.....	41
2.5. BASIC STRENGTH AND ELASTIC PROPERTIES OF PLYWOOD.....	41
2.50. General.....	41
2.51. Analysis of Plywood Strength Properties.....	42
2.52. Basic Formulas.....	42
2.53. Approximate Methods for Calculating Plywood Strengths.....	45
2.54. Moisture-strength Relations for Plywood.....	45
2.540. General.....	45
2.541. Approximate Methods for Making Moisture Corrections for Plywood Strength Properties.....	47
2.5410. Moisture Corrections for Plywood Compressive Strength (0° or 90° to Face Grain Direction).....	47
2.5411. Moisture Correction for Plywood Tensile Strength (0° or 90° to Face Grain Direction).....	47
2.5412. Moisture Corrections for Plywood Shear Strength (0° or 90° to Face Grain Direction).....	47
2.5413. Moisture Corrections for Plywood Compressive Strength (Any Angle to Face Grain Direction).....	47
2.5414. Moisture Corrections for Plywood Tensile Strength (Any Angle to Face Grain Direction).....	47
2.5415. Moisture Corrections for Plywood Shear Strength (Any Angle to Face Grain Direction).....	47
2.55. Specific Gravity-strength Relations for Plywood.....	47
2.56. Stress-strain Relations for Wood and Plywood.....	55
2.560. Mohr's Stress-and-strain Circles.....	59
2.5600. Obtaining Strains from Given Stresses.....	59
2.5601. Obtaining Stresses from Given Strains.....	59
2.5602. Experimental Stress-strain Data.....	59
2.57. Stress Concentrations (See Sec. 2.16 to 2.162).....	59
2.6. PLYWOOD STRUCTURAL ELEMENTS.....	59
2.60. Elements ($\theta=0^\circ$ or 90°).....	59
2.600. Elements in Compression ($\theta=0^\circ$ or 90°).....	60
2.601. Elements in Tension ($\theta=0^\circ$ or 90°).....	61
2.602. Elements in Shear ($\theta=0^\circ$ or 90°).....	61
2.61. Elements (θ =Any Angle).....	61
2.610. Elements in Compression (θ =Any Angle).....	61
2.611. Elements in Tension (θ =Any Angle).....	64
2.612. Elements in Shear (θ =Any Angle).....	64
2.613. Elements in Combined Compression (or Tension) and Shear (θ =Any Angle).....	64
2.614. Elements in Bending.....	65
2.6140. Deflections.....	66
2.615. Elements as Columns.....	67
2.7. FLAT RECTANGULAR PLYWOOD PANELS.....	67
2.71. Buckling Criteria.....	67
2.711. General.....	67
2.712. Compression ($\beta=0^\circ$ or 90°).....	68

	Page
2.713. Shear ($\beta=0^\circ$ or 90°).....	69
2.714. Combined Compression and Shear ($\beta=0^\circ$ or 90°).....	69
2.715. Compression, Shear, and Combined Compression and Shear ($\beta=\text{Any Angle}$).....	69
2.7151. Combined Compression (or Tension) and Shear.....	82
2.72. Strength After Buckling.....	92
2.721. General.....	92
2.722. Compression ($\beta=\text{Any Angle}$).....	92
2.723. Shear ($\beta=0^\circ$, 45° , or 90°).....	92
2.73. Allowable Shear in Plywood Webs.....	92
2.730. General.....	92
2.731. Allowable Shear Stresses.....	96
2.732. Buckling of Plywood Shear Webs.....	96
2.74. Lightening Holes.....	96
2.75. Torsional Strength and Rigidity of Box Spars.....	96
2.76. Plywood Panels under Normal Loads.....	96
2.760. General.....	96
2.761. Small Deflections.....	96
2.762. Large Deflections.....	98
2.77. Stiffened Flat Plywood Panels.....	98
2.771. The Stiffness of a Stiffener Affixed to a Plywood Panel.....	98
2.772. A Single Stiffener Bisecting a Panel.....	99
2.7721. Stiffened Panel Subjected to Edgewise Compression, Stiffener Perpendicular to the Direction of the Stress and Parallel to Side A ($\beta=0^\circ$ or 90°).....	99
2.7722. Stiffened Panel Subjected to Edgewise Compression, Stiffener Perpendicular to the Direction of the Stress ($\beta=45^\circ$).....	101
2.7723. Stiffened Panel Subjected to Edgewise Compression, Stiffener Parallel to the Direction of the Stress and to Side B ($\beta=0^\circ$ or 90°).....	101
2.7724. Stiffened Panel Subjected to Edgewise Compression, Stiffener Parallel to the Direction of the Stress ($\beta=45^\circ$).....	101
2.7725. Stiffened Panel Subjected to Edgewise Shear Stiffener Parallel to Edges (or Ends) of Panel and $\beta=0^\circ$ or 90°	101
2.773. A Plywood Panel Stiffened with A multiplicity of Closely Spaced Stiffeners Parallel to One of Its Edges ($\beta=0^\circ$ or 90°).....	101
2.7731. Determination of D_{we}	103
2.7732. Determination of D_{ze}	103
2.7733. Determination of D_{wze}	103
2.7734. Determination of μ_{fzwe}	103
2.774. Stiffened Plywood Panels Subjected to Bending in the Direction of the Stiffeners.....	103
2.775. Modes of Failure in Stiffened Panels.....	107
2.8. CURVED PLYWOOD PANELS.....	107
2.81. Strength in Compression or Shear; or Combined Compression (or Tension) and Shear.....	107
2.82. Circular Thin-walled Plywood Cylinders.....	107
2.821. Axial Compression.....	107
2.8211. Compression with Face Grain Parallel or Perpendicular to the Axis of the Cylinder.....	107
2.8212. Compression with 45° Face Grain.....	109
2.8213. Compression—Effect of Length.....	109
2.822. Bending.....	109
2.823. Torsion.....	109
2.824. Combined Torsion and Bending.....	109
2.83. Curved Panels.....	109
2.831. Axial Compression.....	109
2.832. Shear.....	109
2.84. Longitudinally Stiffened Cylinders.....	109
2.841. Stresses When Buckling Does Not Occur.....	109
2.8411. Axial Compression (or Tension).....	109
2.8412. Shear Stress Due to Torsion.....	111
2.842. Buckling of Stiffened Cylinders.....	112
2.8421. Axial Compression.....	112
2.8422. Torsion.....	112
2.8423. Bending.....	112
2.85. Stiffened Curved Panels.....	112
2.851. Axial Compression.....	112

2.8511. Stiffener Axial.....	112
2.8512. Stiffener Circumferential.....	112
2.852. Shear.....	116
2.8521. Stiffener Axial.....	116
2.9. JOINTS.....	116
2.90. Bolted Joints.....	116
2.900. Bearing Parallel or Perpendicular to Grain.....	116
2.901. Bearing at an Angle to the Grain.....	116
2.902. Eccentric Loading.....	116
2.903. Combined Concentric and Eccentric Loadings; Bolt Groups.....	116
2.904. Bolt Spacings.....	118
2.9040. Spacing of Bolts Loaded Parallel to the Grain.....	118
2.9041. Spacing of Bolts Loaded Perpendicular to the Grain.....	122
2.9042. Spacing of Bolts Loaded at an Angle to the Grain.....	122
2.9043. General Notes on Bolt Spacing.....	122
2.905. Bearing in Wood-base Materials.....	122
2.9050. Bearing in Plywood.....	122
2.9051. Bearing in Compreg.....	123
2.906. Bearing in Reinforced Members.....	125
2.9060. Wood Members with Plywood Reinforcing Plates.....	125
2.9061. Wood Members with Cross-banded Compreg Reinforcing Plates.....	125
2.907. Bushings.....	126
2.908. Hollow Bolts.....	126
2.909. General Features of Bolted Joints.....	126
2.9090. Drilling of Holes.....	126
2.9091. Repeated Loading of Bolted Joints.....	126
2.91. Glued Joints.....	126
2.910. Allowable Stress for Glued Joints.....	126
2.911. Laminated and Spliced Spars and Spar Flanges.....	127
2.912. Glue Stress Between Web and Flange.....	127
2.92. Properties of Modified Wood.....	127
2.920. Detailed Test Data for Tables 2-16 to 2-21, Inclusive.....	127
2.3. References.....	141
2.4. Bibliography.....	144

CHAPTER 3. METHODS OF STRUCTURAL ANALYSIS

3.0. GENERAL.....	146
3.00. Purpose.....	146
3.01. Special Considerations in Static Testing of Structures.....	146
3.010. Element Tests.....	146
3.011. Complete Structures.....	147
3.0110. Design Allowances for Test Conditions.....	147
3.0111. Test Procedure.....	147
3.1. WINGS.....	148
3.10. General.....	148
3.11. Two-spar Wings with Independent Spars.....	148
3.110. Spar Loadings.....	148
3.111. Chord Loading.....	149
3.112. Lift-truss Analysis.....	150
3.1120. General.....	150
3.1121. Lift Struts.....	152
3.1122. Jury Struts.....	154
3.1123. Nonparallel Wires.....	154
3.1124. Biplane Lift Trusses.....	154
3.1125. Rigging Loads.....	154
3.113. Drag-truss Analysis.....	154
3.1130. Single Drag-truss Systems.....	154
3.1131. Double Drag-truss Systems.....	155
3.1132. Fixity of Drag Struts.....	155
3.1133. Plywood Drag-truss Systems.....	155
3.114. Spar Shears and Moments.....	155
3.1140. Beam-column effects (Secondary bending).....	157

	Page
3.1141. Effects of Varying Axial Load and Moment of Inertia.....	158
3.115. Internal and Allowable Stresses for Spars.....	158
3.1150. General.....	158
3.1151. Wood Spars.....	158
3.116. Special Problems in the Analysis of Two-spar Wings.....	159
3.1160. Lateral Buckling of Spars.....	159
3.1161. Ribs.....	159
3.1162. Fabric Attachment.....	159
3.12. Two-spar Plywood Covered Wings.....	160
3.120. Single Covering.....	160
3.121. Box Type.....	160
3.13. Reinforced Shell Wings.....	160
3.130. General.....	160
3.131. Computation of Loading Curves.....	160
3.1310. Loading Axis.....	160
3.1311. Loading Formulas.....	162
3.132. Computation of Shear, Bending Moment, and Torsion.....	163
3.133. Computation of Bending Stresses.....	163
3.1330. Section Properties.....	165
3.1331. Bending Stress Formulas.....	167
3.134. Secondary Stresses in Bending Elements.....	168
3.135. Computation of Shear Flows and Stresses.....	168
3.1350. General.....	168
3.1351. Shear Flow Absorbed by Bending Elements.....	170
3.1352. Shear Correction for Beam Taper.....	172
3.1353. Simple D Spar.....	172
3.1354. Rational Shear Distribution.....	173
3.13540. Single Cell—General Method.....	173
3.13541. Two Cell—General Method.....	176
3.13542. Two-Cell Four-Flange Wing.....	178
3.13543. Shear Centers.....	180
3.136. Ribs and Bulkheads.....	181
3.1360. Normal Ribs.....	181
3.13600. Rib-Crushing Loads.....	181
3.1361. Bulkhead Ribs.....	181
3.137. Miscellaneous Structural Problems.....	182
3.1370. Additional Bending and Shear Stresses due to Torsion.....	182
3.1371. General Instability.....	182
3.138. Strength Determination.....	182
3.1380. Buckling in Skin.....	182
3.1381. Compression Elements.....	183
3.1382. Stiffened Panels.....	183
3.1383. Tension Elements.....	184
3.1384. Shear Elements.....	184
3.2. FIXED TAIL SURFACES.....	185
3.3. MOVABLE CONTROL SURFACES.....	185
3.4. FUSELAGES.....	186
3.40. General.....	186
3.41. Four-longeron Type.....	186
3.42. Reinforced-shell Type.....	189
3.421. Stressed-skin Fuselages.....	189
3.422. Computation of Bending Stresses.....	189
3.423. Computation of Shearing Stresses.....	191
3.43. Pure-shell Type.....	191
3.431. Monocoque shell Fuselages.....	191
3.44. Miscellaneous Fuselage Analysis Problems.....	192
3.441. Analysis of Seams.....	192
3.442. Analysis of Frames and Rings.....	192
3.4421. Main Frames.....	192
3.4422. Intermediate Frames.....	193
3.443. Effects of Cut-Outs.....	193
3.444. Secondary Structures Within the Fuselage.....	193

CHAPTER 3. METHODS OF STRUCTURAL ANALYSIS—Continued

3.45. Strength Determination.....	Page
3.5. HULLS AND FLOATS.....	195
3.51. Main Longitudinal Girder.....	195
3.52. Bottom Plating.....	195
3.53. Bottom Stringers.....	196
3.54. Frames.....	196
3.55. Strength Determination.....	197
3.6. MISCELLANEOUS.....	197
References.....	198

CHAPTER 4. DETAIL STRUCTURAL DESIGN

4.0. GENERAL.....	199
4.00. Introduction.....	199
4.01. Definitions.....	199
4.010. Solid Wood.....	199
4.011. Laminated Wood.....	199
4.012. Plywood.....	199
4.013. High-density material.....	199
4.1. PLYWOOD COVERING.....	199
4.10. General.....	199
4.11. Joints in the Covering.....	199
4.12. Taper in Thickness of the Covering.....	199
4.13. Behavior Under Tension Loads.....	201
4.14. Behavior Under Shear Loads.....	202
4.15. Plywood Panel Size.....	202
4.16. Cut-Outs.....	203
4.2. BEAMS.....	203
4.20. Types of Beams.....	203
4.21. Laminating of Beams and Beam Flanges.....	205
4.22. Shear Webs.....	205
4.23. Beam Stiffeners.....	206
4.24. Blocking.....	206
4.25. Scarf Joints in Beams.....	207
4.26. Reinforcement of Sloping Grain.....	207
4.3. RIBS.....	207
4.30. Types of Ribs.....	207
4.31. Special Purpose Ribs.....	209
4.32. Attachment of Ribs to the Structure.....	209
4.4. FRAMES AND BULKHEADS.....	210
4.40. Types of Frames and Bulkheads.....	210
4.41. Glue Area for Attachment of Plywood Covering.....	210
4.42. Reinforcement for Concentrated Loads.....	210
4.5. STIFFENERS.....	211
4.50. General.....	211
4.51. Attachment of Stringers.....	211
4.52. Attachment of Intercostals.....	211
4.6. GLUE JOINTS.....	212
4.60. General.....	212
4.61. Eccentricities.....	212
4.62. Avoidance of End-Grain Joints.....	212
4.63. Gluing of Plywood Over Wood-Plywood Combinations.....	212
4.64. Gluing of High Density Material.....	213
4.7. MECHANICAL JOINTS.....	214
4.70. General.....	214
4.71. Use of Bushings.....	214
4.72. Use of High Density Material.....	214
4.73. Mechanical Attachment of Ribs.....	215
4.74. Attachment of Various Types of Fittings.....	215
4.75. Use of Wood Screws, Rivets, Nails, and Self-Locking Nuts.....	215
4.8. MISCELLANEOUS DESIGN DETAILS.....	216
4.80. Metal to Wood Connections.....	216

	Page
4.81. Stress Concentrations.....	217
4.82. Behavior of Dissimilar Materials Working Together.....	217
4.83. Effects of Shrinkage.....	217
4.84. Drainage and Ventilation.....	217
4.85. Internal Finishing.....	219
4.86. External Finishing.....	219
4.87. Selection of Species.....	220
4.88. Use of Standard Plywood.....	220
4.89. Tests.....	220
4.9. EXAMPLES OF ACTUAL DESIGN DETAILS.....	220

CHAPTER I

GENERAL

1.0. Purpose and Use of Bulletin

1.00. INTRODUCTION. This bulletin has been prepared for use in the design of both military and commercial aircraft, and contains material which is acceptable to the U. S. Air Force, Navy Bureau of Aeronautics, and the Civil Aeronautics Administration. It should, of course, be understood that methods and procedures other than those outlined herein are also acceptable, provided they are properly substantiated and approved by the appropriate agency. The applicability and interpretation of the provisions of this bulletin as contract or certification requirements will in each case be defined by the procuring or certifying agency.

1.01. SCOPE OF BULLETIN. The technical material in this bulletin is contained in chapters 2, 3, and 4, and pertains to three related phases of the structural design of wood aircraft.

Chapter 2 presents information on the strength and elastic properties of structural elements constructed of wood and plywood. This information supersedes that contained in the June 1944 edition of ANC-18, "Design of Wood Aircraft Structures."

Chapter 3 contains suggested methods of structural analysis for the design of various aircraft components. Although these methods are in many cases the same as those used for metal structures, special considerations have been introduced which take into account the orthotropic properties of wood.

Chapter 4 presents recommendations on the detail structural design of wood aircraft and contains some examples of how various manufacturers have treated the solution of specific detail design problems.

1.02. ACKNOWLEDGMENT. The Panel on Sandwich Construction of the Subcommittee on Air-Force-Navy-Civil Aircraft Design Criteria and the Forest Products Laboratory express their apprecia-

tion to aircraft manufacturers and others for the valuable assistance given in connection with various parts of this bulletin.

1.1. Nomenclature

This section presents the definitions of standard structural symbols which are used in the bulletin. In addition, sections 1.10 and 1.11 are presented to clarify the differentiation between the definitions for strength and elastic properties of plywood elements and those for like properties of plywood panels. These sections also outline the use of table 2-13.

1.10. DEFINITIONS FOR PLYWOOD ELEMENTS—BEAMS, PRISMS, AND COLUMNS IN COMPRESSION, STRIPS IN TENSION. A plywood element is any rectangular piece of plywood that is supported, loaded, or restrained on two opposite edges only. In defining the various strength and elastic property terms for plywood elements the face grain direction has been used as a reference; for example, the subscript w denotes a direction parallel to (with) the face grain, while the subscript x denotes a direction perpendicular to (across) the face grain. This is illustrated by figure 1-1. The strength and elastic properties given in table 2-13 of the bulletin are for plywood elements.

1.11 DEFINITIONS FOR PLYWOOD PANELS. A plywood panel is any rectangular piece of plywood that is supported, loaded, or restrained on more than two edges. In defining the various strength and related property terms for plywood panels, the

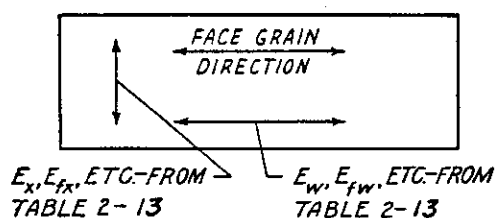
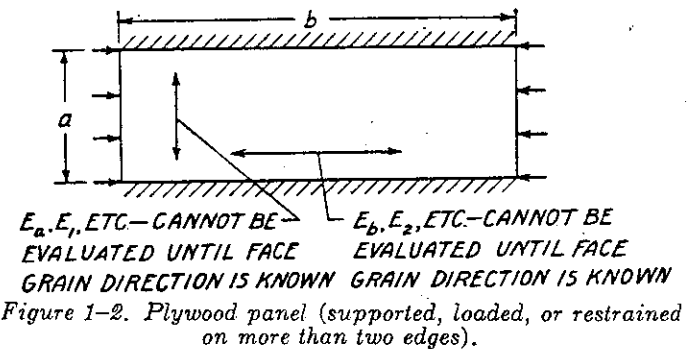


Figure 1-1. Plywood element (supported, loaded, or restrained on two opposite edges only).

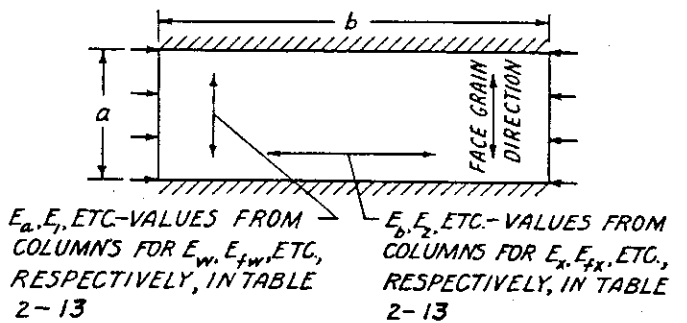
side of length a rather than the face grain direction has been used as the reference. For any panel having tension or compression loads (either alone or accompanied by shear) the side of length a is the loaded side. For panels having only shear loads (with no tension or compression), the side a may be taken as either side (sec. 2.715). For panels having normal loads, side a is the shorter side. The subscripts a and 1 denote a direction parallel to the side of length a , and the subscripts b and 2 denote a direction perpendicular to the side of length a . This is illustrated by figure 1-2.



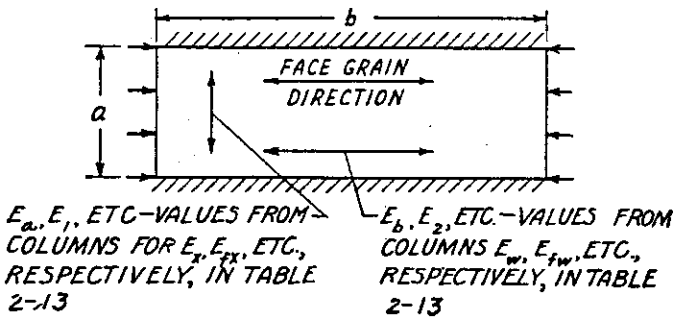
Since, in panels, the directions in which E_a, E_b, E_1, E_2 , etc., are to be measured are related to the directions of the sides of lengths a and b , it is necessary to relate these directions to the face grain direction before the terms can be evaluated from table 2-13. It may be stated, therefore, that:

- (1) When the face grain direction of a plywood panel is parallel to the side of length a , the values of E_a, E_b, E_1, E_2 , etc., may be taken from the columns for

E_w, E_x, E_{fw}, E_{fx} , etc., respectively, in table 2-13. This is illustrated by figure 1-3.



- (2) When the face grain direction of a plywood panel is perpendicular to the side of length a , the values of E_a, E_b, E_1, E_2 , etc., may be taken from the columns for E_x, E_w, E_{fx}, E_{fw} , etc., respectively, in table 2-13. This is illustrated by figure 1-4.



1.12. STANDARD STRUCTURAL SYMBOLS FOR CHAPTER 2. In general, symbols that are used only in the section where they are defined are not included in this nomenclature.

- | | |
|--|--|
| <p>A —area of cross section, square inches (total).</p> <p>A_L —area of plies with grain direction parallel to the direction of applied stress.</p> <p>A_T —area of plies with grain direction perpendicular to the direction of applied stress (surfaces of plies parallel to plane of glue joint tangential to the annual growth rings, as for rotary-cut or flat-sliced veneer, flat-sawn lumber).</p> | <p>a —the length of the loaded side of a plywood panel for compression or tension loads, and the length of either side for shear loads (sec. 2.715); subscript denoting parallel to side of length a for plywood panels.</p> |
|--|--|

A_R	—area of plies with grain direction perpendicular to the direction of applied stress (surfaces of plies parallel to plane of glue joint radial to the annual growth rings, as for quarter-sliced veneer, quarter-sawn lumber).		
B		b	—the length of the unloaded side of a plywood panel for compression or tension loads, and the length of either side for shear loads (sec. 2.715); subscript denoting parallel to side of length b for plywood panels; subscript denoting “bending” for solid wood.
C	—circumference.	b_r	—subscript denoting “bearing.”
		c	—end-fixity coefficient for columns; subscript denoting “compression”; distance from neutral axis to extreme fiber.
		c'	—distance from neutral axis to the extreme fiber having grain direction parallel to the applied stress (plywood).
		cr	—subscript denoting “critical.”
D	—diameter.	d	—depth or height.
D_w	—depending on direction of stiffeners, $\frac{E_{fw}t^3}{12\lambda_f}$ or $\frac{E_{fx}t^3}{12\lambda_f}$		
D_{wc}	—same as D_w except that the stiffener is considered to be an extra ply of the plywood.		
D_{we}	—same as D_w but these are effective values applicable to the stiffened panel as a whole.		
D_z	—depending on direction of stiffeners, $\frac{E_{fx}t^3}{12\lambda_f}$ or $\frac{E_{fw}t^3}{12\lambda_f}$		
D_{zc}	—same as D_z except that the stiffener is considered to be an extra ply of the plywood.		
D_{ze}	—same as D_z but these are effective values applicable to the stiffened panel as a whole.		
D_{wx}	$\frac{G_{fwx}t^3}{12}$		
D_{wxc}	—same as D_{wx} except that the stiffener is considered to be an extra ply of the plywood.		
D_{wxe}	—same as D_{wx} but these are effective values applicable to the stiffened panel as a whole.		

E_L	—modulus of elasticity of wood in the direction parallel to the grain, as determined from a static bending test. (This value is listed in table 2-4.)	e_L	—unit strain (tension or compression) in the L direction.
E_R	—modulus of elasticity of wood in the direction radial to the annual growth rings.	e_R	—unit strain (tension or compression) in the R direction.
E_T	—modulus of elasticity of wood in the direction tangential to the annual growth rings.	e_T	—unit strain (tension or compression) in the T direction.
E_{Lc}	—modulus of elasticity of wood in the direction parallel to the grain, as determined from a compression test (value <i>not</i> listed in table 2-3, but approximately equal to 1.1 E_L).		
		e_{LT}	—unit strain (shear); the change in angle between lines originally drawn in the L and T directions.
		e_{LR}	—unit strain (shear); the change in angle between lines originally drawn in the L and R directions.
		e_{TR}	—unit strain (shear); the change in angle between lines originally drawn in the T and R directions.
E_a	—effective modulus of elasticity of plywood in tension or compression measured parallel to the side of length a of plywood panels.		
E_b	—effective modulus of elasticity of plywood in tension or compression measured perpendicular to the side of length a of plywood panels.		
E_g	— E_a or E_b as required.		
E_w	—effective modulus of elasticity of plywood in tension or compression measured parallel to (<i>with</i>) the grain direction of the face plies.		
E_x	—effective modulus of elasticity of plywood in tension or compression measured perpendicular to (<i>across</i>) the grain direction of the face plies.		
E_{fw}	—effective modulus of elasticity of plywood in flexure (<i>bending</i>) measured parallel to (<i>with</i>) the grain direction of the face plies.		
E_{fx}	—effective modulus of elasticity of plywood in flexure (<i>bending</i>) measured perpendicular to (<i>across</i>) the grain direction of the face plies.		
E'_{fx}	—same as E_{fx} , except that outermost ply on tension side is neglected (not to be used in deflection formulas.)		

E_s	— E_L of a stiffener.		
E_1	—effective modulus of elasticity of plywood in flexure (<i>bending</i>) measured parallel to the side of length a of plywood panels, or parallel to the axis of plywood cylinders.		
E_2	—effective modulus of elasticity of plywood in flexure (<i>bending</i>) measured perpendicular to the side of length a of plywood panels.		
F	—allowable stress; stress determined from test.	f	—internal (or calculated) stress; subscript denoting "flexure" (<i>bending</i>) for plywood.
F_b	—allowable bending stress.	f_b	—internal (or calculated) primary bending stress.
F_{bu}	—modulus of rupture in bending for solid wood parallel to grain.		
F_{bp}	—fiber stress at proportional limit in bending for solid wood parallel to grain.		
F_{brp}	—bearing stress at proportional limit parallel to the grain for solid wood.	f_{br}	—internal (or calculated) bearing stress.
F_{brT}	—allowable ultimate bearing stress perpendicular to grain for solid wood (either radial or tangential to the annual growth rings).		
F_{bru}	—allowable ultimate bearing stress parallel to grain.		
F_c	—allowable compressive stress.	f_c	—internal (or calculated) compressive stress.
F_{ccr}	—critical compressive stress for the buckling of rectangular plywood panels.	f_{cL}	—internal (or calculated) compressive stress in a longitudinal ply; i. e., any ply with its grain direction parallel to the applied stress.
F_{cp}	—stress at proportional limit in compression parallel to grain for solid wood.		
F_{cpT}	—stress at proportional limit in compression perpendicular to grain for solid wood (either radial or tangential to the annual growth rings).		
F_{cpw}	—stress at proportional limit in compression for plywood having the face grain direction parallel to (<i>with</i>) the applied stress.		
F_{cpx}	—stress at proportional limit in compression for plywood having the face grain direction perpendicular to (<i>across</i>) the applied stress.		

$F_{cp\theta}$	—stress at proportional limit in compression for plywood having the face grain direction at an angle θ to the applied stress.		
F_{cu}	—ultimate compressive stress parallel to the grain for solid wood.		
F_{cuT}	—compressive strength perpendicular to grain for solid wood (either radial or tangential to the annual growth rings) (sec. 2.1000).		
F_{cuw}	—ultimate compressive stress for plywood having the face grain direction parallel to (<i>with</i>) the applied stress.		
F_{cuz}	—ultimate compressive stress for plywood having the face grain direction perpendicular to (<i>across</i>) the applied stress.		
$F_{cu\theta}$	—ultimate compressive stress for plywood having the face grain direction at an angle θ to the applied stress.		
F_s	—allowable shearing stress.	f_s	—internal (or calculated) shearing stress.
F_{scr}	—critical shear stress for the buckling of rectangular plywood panels.		
F_{st}	—modulus of rupture in torsion.		
F_{su}	—ultimate shear stress parallel to grain for solid wood.		
$F_{s\theta c}$	—ultimate shear stress for plywood, wherein θ designates the angle between the face grain direction and the shear stress in a plywood element so loaded in shear that the face grain is stressed in compression.		
$F_{s\theta t}$	—ultimate shear stress for plywood, wherein θ designates the angle between the face grain direction and the shear stress in a plywood element so loaded in shear that the face grain is stressed in tension.		
F_{suw}	—ultimate shear stress for plywood elements for the case where the face grain is at 0° and 90° to the shear stress.		
F_t	—allowable tension stress.	f_t	—internal (or calculated) tensile stress.
		f_{tL}	—internal (or calculated) tensile stress in a longitudinal ply (any ply with its grain direction parallel to the applied stress).
F_{tu}	—ultimate tensile stress parallel to grain for solid wood.		
F_{tuT}	—tensile strength perpendicular to grain for solid wood (either radial or tangential to the annual growth rings).		

F_{tuw}	—ultimate tensile stress for plywood having the face grain direction parallel to (with) the applied stress.		
F_{tuz}	—ultimate tensile stress for plywood having the face grain direction perpendicular to (across) the applied stress.		
$F_{tu\theta}$	—ultimate tensile stress for plywood having the face grain direction at an angle θ to the applied stress.		
G	—mean modulus of rigidity taken as $\frac{1}{6}$ of E_L .	g	—length of side of panel and equal to a or b as is required.
G_{LT}	—modulus of rigidity associated with shear deformations in the LT plane resulting from shear stresses in the LR and RT planes.		
G_{LR}	—modulus of rigidity associated with shear deformations in the LR plane resulting from shear stresses in the LT and RT planes.		
G_{TR}	—modulus of rigidity associated with shear deformations in the TR plane resulting from shear stresses in the LT and LR planes.		
H	—a constant, generally theoretical.	h	—height or depth.
I	—moment of inertia.	i	—subscript denoting “ i^{th} ply.”
I_p	—polar moment of inertia.	j	—stiffness factor $\sqrt{EI/P}$
J	—torsion constant (I_p for round tubes).	k	—stiffness factor
K	—a constant, generally empirical.		$\frac{E_{fw}\mu_{fzw} + 2\lambda_f G_{fuz}}{\sqrt{E_{fw}E_{fz}}}$
L	—length; span; subscript denoting the direction parallel to the grain.	l	—not used, to avoid confusion with the numeral 1.
L'	$= \frac{L}{\sqrt{c}}$ where c is the end-fixity coefficient.		
M	—applied bending moment.	m	—number of half waves.
N	—	n	—number of plies, number of stiffeners.
P	—applied load (total, not unit load).	p	—subscript denoting “polar”; subscript denoting “proportional limit”; load per unit area.
		psi	—pounds per square inch.
Q	—static moment of a cross section.	q	—shear flow, pounds per inch.
R	—subscript denoting the direction radial to the annual growth rings and perpendicular to the grain direction.	r	—radius; adjusted ratio length to width
S	—shear force.		$\frac{b}{a} \left(\frac{E_1}{E_2} \right)^{1/4}$
T	—applied torsional moment, torque; subscript denoting the direction tangential to the annual growth rings and perpendicular to the grain direction.	s	—distance c to c of adjacent stiffeners; subscript denoting “shear”.
		t	—thickness; subscript denoting “tension”.
		t_c	—thickness of central ply.
		t_f	—thickness of face ply.

U	—	u	—subscript denoting “ultimate”.
W	—	w	—deflection of plywood panels; load per linear inch; subscript denoting parallel to face grain of plywood.
X	—	x	—subscript denoting perpendicular to face grain of plywood.
Y	—	y	—distance from the neutral axis to any given fiber.
Z	—section modulus, I/c	z	—
Z_p	—polar section modulus, I_p/c .	z_n	—distance between center of panel and neutral axis of panel stiffener combination.
		β	—the angle between side of length b and the face grain direction as used in the determination of buckling criteria for panels (sec. 2.71).
		δ	—deflection.
		θ	—usually the acute angle in degrees between the face grain direction and the direction of the applied stress; angle of twist in radians in a length (L).
		μ_{LT}	—Poisson's ratio of contraction along the direction T to extension along the direction L due to a normal tensile stress on the RT plane; similarly, μ_{LR} , μ_{RT} , μ_{TR} , μ_{RL} , and μ_{TL} .
		ρ	—radius of gyration.
		ϕ	—usually the acute angle in degrees between the face grain direction and the axis of extension.

CHAPTER 2

STRENGTH OF WOOD AND PLYWOOD ELEMENTS

2.0. Physical Characteristics and Factors Affecting the Strength of Wood

2.00. ANISOTROPY OF WOOD. Wood, unlike most other commonly used structural materials, is not isotropic. It is a complex structural material, consisting essentially of fibers of cellulose cemented together by lignin. It is the shape, size, and arrangement of these fibers, together with their physical and chemical composition that govern the strength of wood, and account for the large difference in properties along and across the grain (ref. 2-20).

The fibers are long and hollow tubes tapering toward the ends, which are closed. Besides these vertical fibers, which are oriented with their longer dimension lengthwise of the tree and comprise the principal part of what is called wood, all species, except palms and yuccas, contain horizontal strips of cells known as rays, which are oriented radially and are an important part of the tree's food transfer and storage system. Among different species the rays differ widely in their size and prevalence.

From the strength standpoint, this arrangement of fibers results in an anisotropic structure, that accounts for three Young's moduli differing by as much as 150 to 1, three shear moduli differing by as much as 20 to 1, six Poisson's ratios differing by as much as 40 to 1, and other properties differing by various amounts. Not all of these wood properties have, as yet, been thoroughly evaluated.

Figure 2-1 shows a diagrammatic sketch of the cellular structure of wood. Each year's growth is represented by one annual ring. The portion of the growth occurring in the spring consists of relatively thin-walled fibers, while that occurring during the later portion of the growing season consists of fibers having somewhat heavier walls. Thus, there is, for most woods, a definite line of

demarcation between the growth occurring in successive years. The relation between the cellular structure of the wood and the three principal axes—longitudinal (L), tangential (T), and radial (R)—is indicated on the sketch. Figure 2-2 shows the relation between these axes and (a) the log, (b) a flat-sawn board or rotary-cut veneer, and (c) an edge-grain board or quarter-sliced veneer.

2.01. DENSITY OR APPARENT SPECIFIC GRAVITY. The substance of which wood is composed is actually heavier than water, its specific gravity being nearly the same for all species and averaging about 1.5. Since a certain proportion of the volume of wood is occupied by cell cavities, the apparent specific gravity of the wood of most species is less than unity.

Relations between various strength properties and specific gravity have been developed (table 2-1) and are useful in estimating the strength of a piece of wood of known specific gravity. Considerable variability from these general relations is found, so that while they cannot be expected to give exact strength values, they do give good estimates of strength. Minimum permissible specific gravity values are listed in section 2.10.

The exponential values shown in table 2-1 apply to variation within a species. That is, they are to be used in determining the relation between the strength properties of pieces of the same species but of different specific gravity. For expressing the relation between the average strength properties of different species, the exponential values are somewhat lower. Such values are shown in table 14 of U. S. Department of Agriculture Technical Bulletin 479 (ref. 2-57).

2.02. MOISTURE CONTENT. Wood in the natural state in the living tree has considerable water associated with it. After being converted to lumber or other usable form, or during conversion,

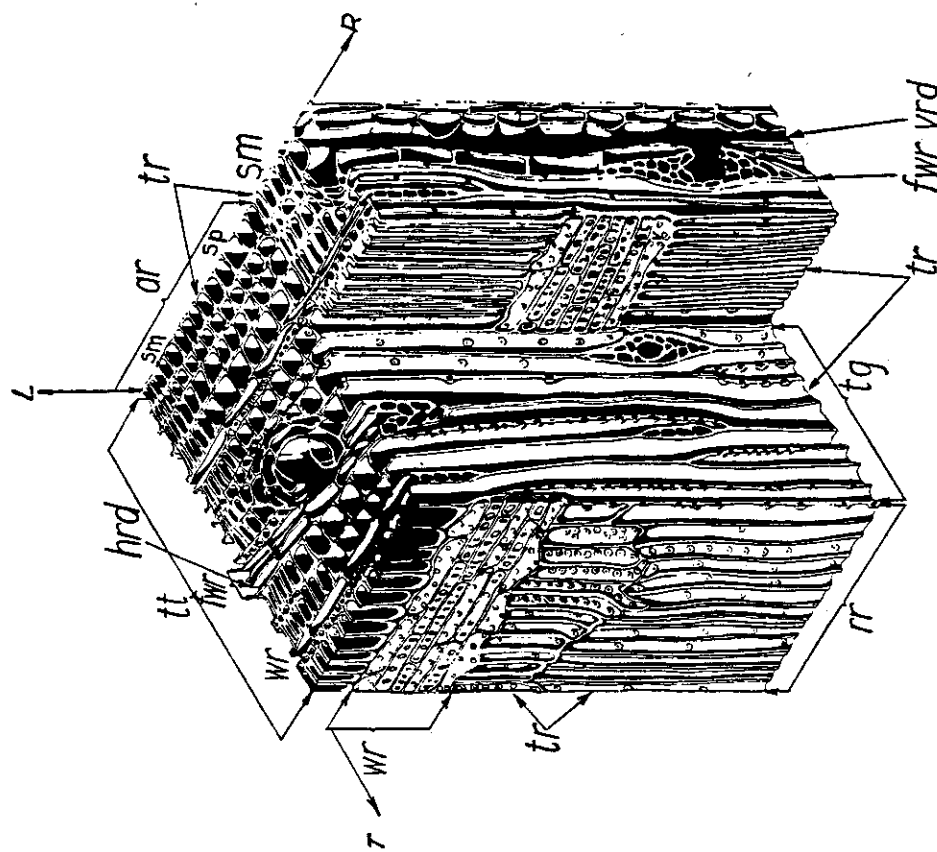


Figure 2-1. Wood cellular structure. Drawing of a highly magnified block softwood measuring about one-fortieth inch vertically: tt, transverse surface; rr, radial surface; tg, tangential surface; ar, annual rings; sm, summerwood; sp, springwood; tr, tracheids, or fibers; hrd, horizontal resin duct; fur, fusiform wood ray; wr, wood rays; L, direction (longitudinal) of grain; R, direction radial to annual rings and perpendicular to grain direction; T, direction tangential to annual rings and perpendicular to grain direction; vrd, vertical resin duct.

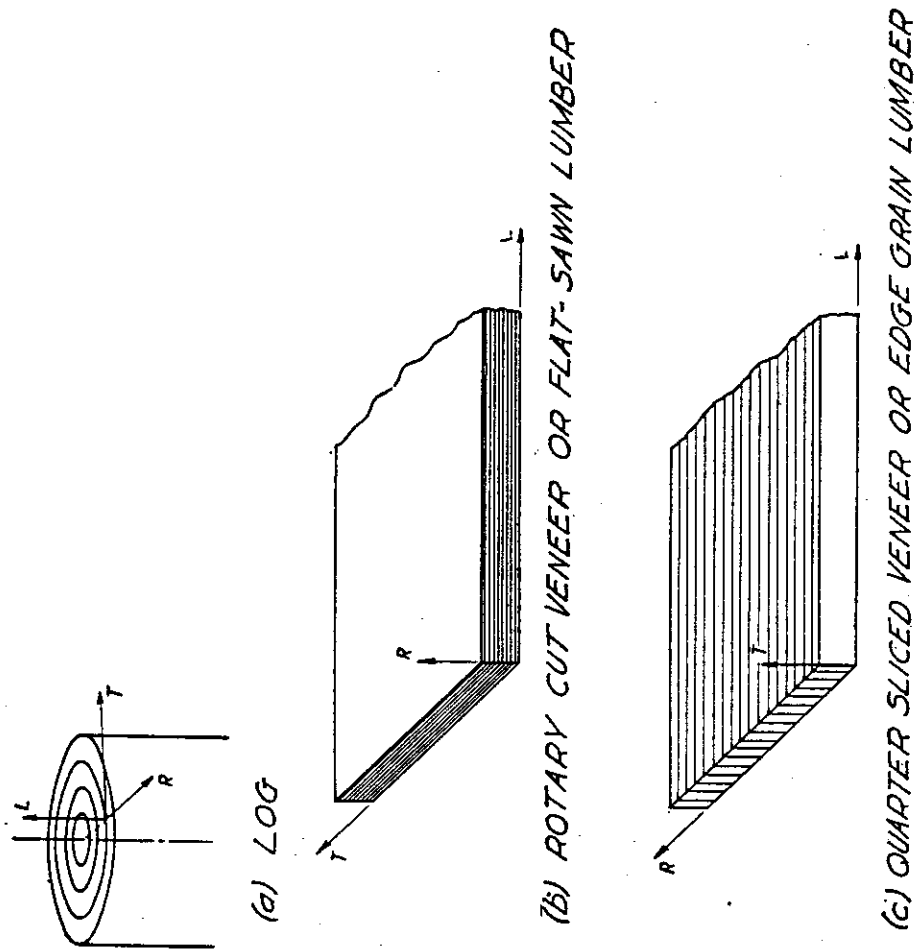


Figure 2-2. Principal directions in wood and plywood.

wood is commonly dried so that most of the water is removed.

The water is associated with the wood in two ways, either absorbed in the cell walls, or as free water in the cell cavities. During drying, the free water in the cell cavities is removed first, then that absorbed in the cell walls. The point at which all the water has been removed from the cell cavities while the cell walls remain saturated is known as the fiber-saturation point. For most species, the moisture content at fiber saturation is from 22 to 30 percent of the weight of the dry wood.

Lowering the moisture content to the fiber-saturation point results in no changes in dimension or in strength properties. Lowering the moisture content below the fiber-saturation point, however, results in shrinkage and an increase in strength properties.

Wood is a hygroscopic material, continually giving off or taking on moisture in accordance with the relative humidity and temperature to which it is exposed. Thus, while the strength of a piece of wood may be increased to a relatively high value by drying to a low moisture content, some of that increase may be lost if, in use, it is exposed to atmospheric conditions that tend to increase the moisture content. While paint and other coatings may be employed to retard the rate of absorption of moisture by wood, they do not change its hygroscopic properties, thus a piece of wood may be expected to come to the same moisture content under the same exposure conditions whether painted or unpainted. The time required will vary, depending upon whether or not it is coated. It is desirable, therefore, to design a structure on the basis of the strength corresponding to the conditions of use.

Moisture content is generally expressed as a percentage of the dry weight of the wood. The percentage variation of wood strength properties for 1 percent change in moisture content is given in table 2-2. Since this variation is an exponential function (ref. 2-85), it is necessary that strength adjustments based on the percentage changes given in the table be made successively for each 1 percent change in moisture content until the total change has been covered. Figure 2-3 is a chart by means of which the ratio between the adjusted strength and the original strength may be determined approximately if the proper correction factor is obtained from table 2-2 and the difference in moisture content for which correction is

desired is known. For positive correction factors in table 2-2, the original strength is multiplied by the strength ratio factor determined from figure 2-3 for adjustments involving decrease in moisture content, and divided by the strength ratio factor for those involving increase in moisture content. For negative factors in table 2-2, the original strength is divided by the strength ratio factor for adjustments involving decrease in moisture, and multiplied for adjustments involving increase in moisture.

Table 2-1. Variation of wood strength properties with specific gravity ¹

$$\frac{S}{S'} = \left(\frac{g}{g'}\right)^n$$

S = strength at specific gravity g
S' = strength at specific gravity g' (usually average values from column (2) of tables 2-6 and 2-7)

	n
Static bending:	
Fiber stress at proportional limit.....	1.50
Modulus of rupture.....	1.50
Modulus of elasticity.....	1.25
Work to maximum load.....	2.00
Total work.....	2.25
Impact bending:	
Fiber stress at proportional limit.....	1.50
Modulus of elasticity.....	1.25
Height of drop.....	2.00
Compression parallel to grain:	
Fiber stress at proportional limit.....	1.25
Maximum crushing strength.....	1.25
Modulus of elasticity.....	1.25
Compression perpendicular to grain:	
Fiber stress at proportional limit.....	2.50
Hardness—end, radial, tangential.....	2.50

¹ Values in this table apply only to variations within a species. See section 2.01.

2.03. SHRINKAGE. Reduction of moisture content below the fiber-saturation point results in a change in dimension of the wood. Shrinkage in the longitudinal direction is generally negligible, but in the other two directions it is considerable. In general, radial shrinkage is less than tangential, the ratio between the two varying with the species.

A quarter-sawed board will, therefore, shrink less in width but more in thickness than a flat-sawed board. The smaller the ratio of radial to tangential shrinkage, the more advantage is to be gained through minimizing shrinkage in width by

using a quarter-sawed board. The smaller the difference between radial and tangential shrinkage, the less, ordinarily, is the tendency to check in drying and to cup with changes in moisture content.

In general, woods of high specific gravity shrink

and swell more for a given change in moisture content than do woods of low specific gravity.

2.04. TEMPERATURE. The strength of wood is greatly influenced by its temperature, but the magnitude of the effect depends upon the moisture content of the wood and the time of exposure.

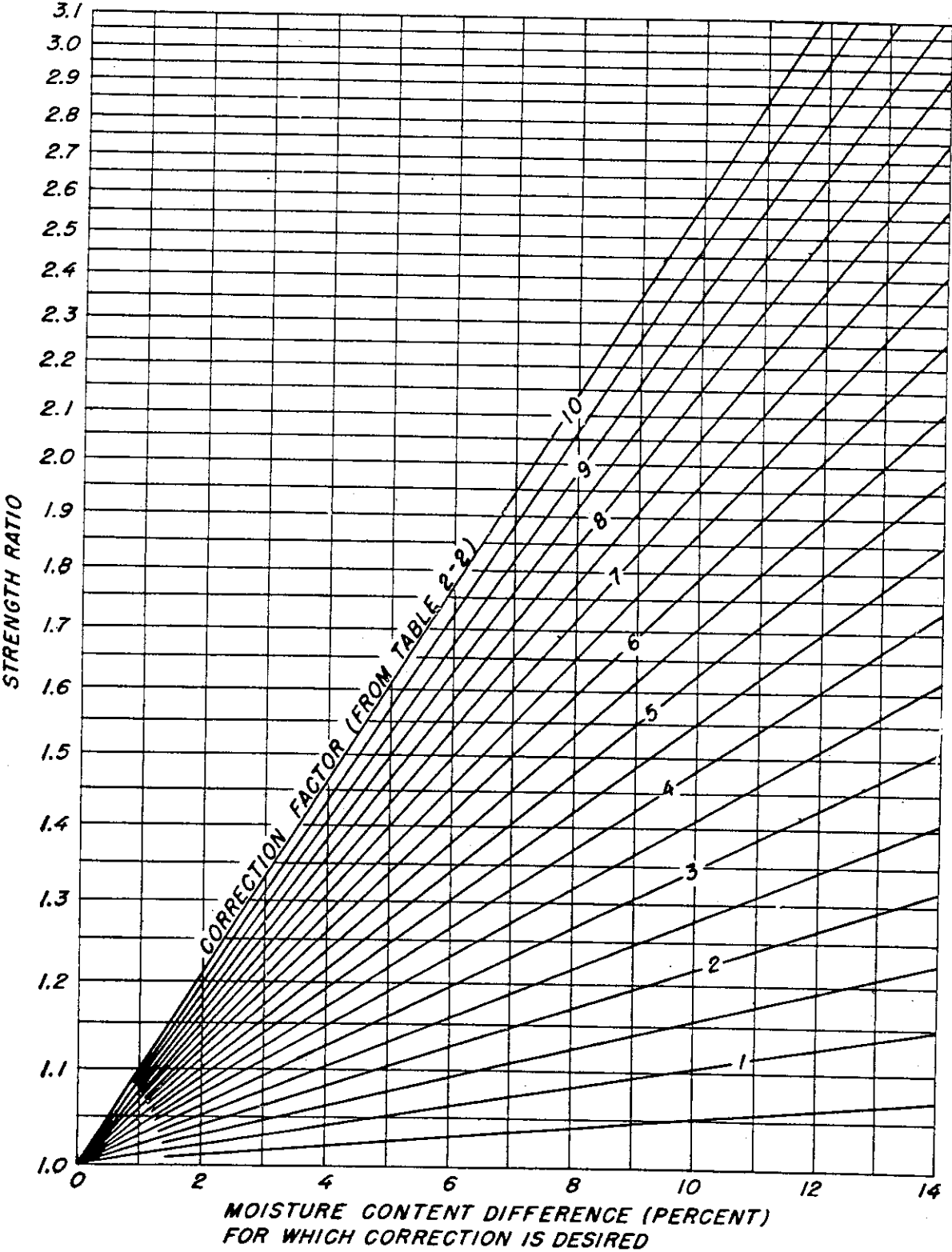


Figure 2-3. Strength ratio chart for use in making strength-moisture adjustments.

Table 2-2. Percentage increase in wood strength properties for 1 percent decrease in moisture content ¹

Species	Static bending				Compression parallel to grain, maximum crushing strength	Compression perpendicular to grain	Shearing strength parallel to grain	Hardness (side)
	Fiber stress at proportional limit	Modulus of rupture	Modulus of elasticity	Work to maximum load ²				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Hardwoods: ³								
Ash, black	8.9	6.4	3.6	1.8	8.3	6.8	5.1	4.1
Ash, commercial white	4.1	3.5	1.4	.4	4.7	4.8	2.9	2.4
Basswood, American	6.8	4.8	2.9	2.6	6.5	6.6	4.2	4.2
Beech, American	6.0	4.7	1.8	2.0	6.2	5.3	3.8	3.6
Birch, sweet	6.4	5.0	2.3	1.2	7.1	7.2	5.0	3.6
Birch, yellow	6.0	4.8	2.0	1.7	6.1	5.6	3.6	3.3
Cherry, black	6.6	3.6	1.1	1.0	6.0	5.5	3.5	3.1
Cottonwood	5.8	4.1	2.5	.1	6.6	5.7	2.6	1.8
Elm, rock	4.7	3.8	2.1	-.3	5.3	6.1	3.5	2.8
Hickory (true hickories)	4.9	4.8	2.8	-.7	5.9	6.6	3.9	
Khaya ("African mahogany")	3.2	2.5	1.6	-.6	3.2	3.0	.4	3.1
Mahogany	2.6	1.3	.8	-2.9	2.5	3.9		1.0
Maple, sugar	5.2	4.4	1.4	1.9	5.7	7.1	3.9	3.4
Oak, commercial white and red	4.6	4.4	2.4	1.7	5.9	4.4	3.5	1.8
Sweetgum	6.7	4.7	2.2	1.5	6.1	5.4	3.5	2.4
Walnut, black	5.8	3.7	1.4	-2.6	4.8	6.3	1.0	1.0
Yellow-poplar	5.0	4.6	2.7	1.9	6.7	4.8	3.3	2.4
Softwoods (conifers): ³								
Baldcypress	4.6	4.0	1.6	1.8	4.9	5.1	1.7	2.3
Douglas-fir	4.5	3.7	1.8	1.9	5.5	5.0	1.7	2.9
Fir, noble	5.1	4.7	1.9	3.2	6.1	5.5	2.3	3.1
Hemlock, western	4.7	3.4	1.4	.7	5.0	3.7	2.5	2.0
Incense cedar, California	3.4	2.1	1.8	-1.4	4.3	4.0	.4	1.5
Pine, eastern white	5.6	4.8	2.0	2.1	5.7	5.6	2.2	2.2
Pine, red	8.0	5.7	2.2	4.7	7.5	7.2	3.9	4.5
Pine, sugar	4.4	3.9	2.1	.1	5.4	4.4	3.7	1.9
Pine, western white	5.3	5.1	2.2	4.8	6.5	5.2	2.5	1.5
Redcedar, western	4.3	3.4	1.6	1.3	5.1	5.1	1.6	2.3
Spruce, red and Sitka	4.7	3.9	1.7	2.0	5.3	4.3	2.6	2.4
Spruce, white	5.8	4.8	1.9	2.1	6.5	5.7	3.7	3.3
White-cedar, northern	5.4	3.6	1.8	-1.5	5.9	2.3	2.8	3.0
White-cedar, Port Orford	5.7	5.2	1.6	1.7	6.2	6.7	2.2	2.8

¹ Corrections to the strength properties should be made successively for each 1 percent change in moisture content until the total change has been covered. For each 1 percent decrease in moisture content, the strength is multiplied by (1+P), where P is the percentage correction factor shown in the table, expressed as a decimal. For each 1 percent increase in moisture content, the strength is divided by (1+P).

² Negative values indicate a decrease in work to maximum load for a decrease in moisture content.

³ For tension values see section 2.5411.

Although considerable literature exists relating to permanent strength reductions resulting from prolonged or cyclic exposure to temperature extremes (ref. 2-46, 2-57, 2-84) few investigations have been made with respect to transient or reversible effects resulting from differences in temperature (ref. 2-2, 2-26, 2-28, 2-36, 2-58, 2-73, 2-76, 2-81). Broadly, it would appear from data now available that prolonged exposure to temperatures

above about 150° F. may result in a permanent loss of strength, and that within the range 0° F. to 150° F. the reduction in static strength properties (excluding modulus of elasticity) with increase in temperature for wood at about 12 percent moisture content, will approximate one-half percent per degree Fahrenheit. The effect on impact properties is variable and cannot be generalized. Some data on the mechanical properties of wood of different moisture contents over the tempera-

ture range of -328°F. to $+392^{\circ}\text{F.}$ are given in NACA Technical Memorandum No. 984, reference 2-37.

2.05. **FATIGUE PROPERTIES.** The fatigue characteristics of wood have been explored to only a limited extent (ref. 2-38, 2-39, 2-41). Tests of the Forest Products Laboratory indicate that wood is less sensitive to rapidly repeated loads than are the more crystalline structural materials, resulting in a higher endurance limit in proportion to the ultimate strength. Tension parallel-to-grain and glue joint shear tests of Douglas-fir and white oak, and tension perpendicular-to-grain tests of Douglas-fir showed an endurance load for 30 million cycles of about 40 percent of standard test strength for both solid wood and scarf-jointed specimens when the minimum repeated load was 10 percent of the maximum repeated load for each cycle. These tests were conducted at about 12 percent moisture content, at a temperature of 75°F.

Tests of small cantilever bending specimens of solid wood and plywood, subjected to fully reversed stresses under the same temperature-humidity conditions as above, indicate an endurance load of about 30 percent of the modulus of rupture of standard static tests after 30 million cycles of repeated stress.

Very little data are available on the effects of notches, bolt holes, or connectors in fatigue. Some data on the effect of notches on fatigue properties of rotating beam specimens are given in reference 2-87.

2.06. **PLASTIC PROPERTIES.** Though it is known that wood, in common with other materials of construction, exhibits plastic as well as elastic properties, quantitative evaluations of the plastic characteristics of wood are limited in scope (ref. 2-34). Thus, while recognizing that when load is applied to a wood structural member the immediate or elastic deformation subsequently will be increased by plastic yield or creep, there is, at present, no accurate means of evaluating the rate of progress of such plastic yield, nor of predicting the time at which failure may be expected to occur.

Preliminary investigations at the Forest Products Laboratory (ref. 2-66, 2-86) involve creep tests with stresses up to 3,000 pounds per square inch in tension and compression, up to 400 pounds per square inch in shear, and at stress levels approaching the short-time ultimate strength in bending. They indicate creep characteristics very

much alike in these strength properties at the lower stress levels. There is reason to believe that creep properties differ between tension and compression as stress values approach the short-time ultimate strength in compression, but the difference has not been fully explored. From these considerations, supported by a few test results, it is believed that joints and fastenings have similar creep properties insofar as their strength is controlled by the strength of the wood.

It is evident from the studies thus far completed that creep under constant load is quite rapid at first, and continues for long periods at a decreasing rate, depending upon the ratio of the applied load to that which would cause immediate failure. No critical point in time has been found at which the rate of creep suddenly changes or at which all creep ceases. Neither has there been found a stress threshold below which no creep takes place.

The studies in bending and compression have indicated that with stresses at, or less than, 60 percent of the strength in a standard laboratory test, the ratio of creep to initial strain is approximately constant. Since strain in this range is proportional to stress, it may be said that creep is proportional to stress. Both in compression and bending, creep at 5 minutes is generally less than 2 percent, and at 1 hour is about 4 percent of the initial strain or deflection. Where stresses in bending exceed 60 percent of laboratory test strength, creep percentages are higher than the above values.

It is known that temperature and moisture changes influence plastic properties, but the extent of such effects is not known.

When load is removed from wood, the elastic strain is recovered immediately. There is, in addition, a recovery of a portion of the plastic strain, relatively rapid at first, and continuing more and more slowly for considerable periods of time. Little is known of the nature and extent of recovery, or of permanent set characteristics, hysteresis, or damping capacity (see sec. 2.095).

In bending, successive repetitions of the same load separated by periods of recovery cause successively increased deformations which finally lead to failure. From data now available it appears that the sum of the periods under intermittent load, until failure occurs, is somewhat greater than the duration to failure under constant load at the same stress level.

2.07. **IMPACT PROPERTIES.** The rate at which

load is applied to a wood member, as well as the time during which it acts, has an important effect on its ability to carry load (secs. 2.06 and 2.10).

Under extremely rapid loading conditions, such as are obtained in impact tests where failure may occur in a fraction of a second, a wood member would be expected to withstand a force much greater than in a standard test. Exact relationships of failing stresses under static and impact loadings are not well known as, in impact tests, measurement is generally made of the absorbed energy rather than the stress imposed.

A beam subjected to impact loading in the standard drop hammer impact bending test (ref. 2-57) may deflect about twice as much before failure as under static test conditions. The stress required to produce failure under such impact conditions is, therefore, correspondingly, about twice as great as for static conditions.

The shock resistance of wood may be measured by a single-blow impact test such as the "toughness" test described in USDA Tech. Bul. 479 (ref. 2-57) where average toughness values for common species are listed, together with minimum acceptance requirements for a number of aircraft woods.

Recent data of the Forest Products Laboratory (ref. 2-10, 2-14) show that the toughness of wood is considerably influenced by its moisture content. Above the fiber-saturation point, approximately 30 percent moisture content, toughness is apparently independent of changes in moisture. For drier material the toughness, in general, decreases slowly from the green value to a minimum at about 12 to 18 percent moisture content, and then increases substantially with further decrease in moisture. These conclusions are in general agreement with extensive toughness and Izod tests by the Australian Forest Products Laboratory (ref. 2-35) but it has been found that results vary greatly among species, and no adequate general formula can be devised to represent toughness-moisture relations for all species.

2.08. EFFECT OF LIQUIDS. Some liquids, when absorbed by wood, adversely affect the strength properties, others do not. In general, nonpolar, nonswelling liquids that do not react with wood chemically do not affect the strength (ref. 2-19). For example, saturated, straight-chain hydrocarbon oils, such as gasoline, kerosene, and most lubricating oils, and aromatic hydrocarbons, such as benzene and toluene, have no significant effect

on the strength of wood and they do not raise the grain.

Turpentine, mineral paint thinners, and linseed oil will by analogy cause practically no effect upon the strength properties nor will cleaning fluids such as carbon tetrachloride.

Low molecular-weight, simple alcohols such as methyl (wood alcohol), ethyl (grain alcohol), and propyl alcohol and polyhydric alcohols such as ethylene glycol (antifreeze) and glycerine swell wood appreciably and cause a considerable loss in strength properties, varying from half to almost the full strength loss caused by water. In general, the crushing strength is decreased somewhat less than it would be if swollen to the same extent in water (ref. 2-19, 2-60).

Lacquer solvents, such as acetone, methyl and ethyl acetates, and ethylene glycol monoethyl ether will reduce the strength about half as much as water.

Low molecular-weight organic acids, such as acetic acid, will reduce the strength about three-fourths as much as water. The high molecular-weight fatty acids will have a much smaller but positive effect.

These various liquids reduce the strength of wood only while they remain in the wood. Volatile liquids, hence, have only a short, temporary effect upon the strength. Low volatility liquids like glycerine will reduce the strength over considerable periods of time. These are apparently the only liquids with which the aircraft industry need be concerned. Some hydraulic fluids containing glycerine or ethylene glycol have an experimentally demonstrated detrimental effect upon the strength of wood (ref. 2-11).

Another group of liquids, strong mineral acids and bases, have a permanent effect upon the strength of woods as a result of chemical degradation of the wood. This degradation varies with species. The only comprehensive data available on the subject are given in the tables 2-3 and 2-4.

2.09. MISCELLANEOUS PHYSICAL PROPERTIES.

2.091. *Coefficient of expansion.* The isotropic nature of wood results in differing coefficients of thermal expansion (α) along its radial, tangential, and fiber axes. This anisotropy, modified by the treatments involved, remains a basic property of all the derived structural products of wood, in which the fiber arrangement is not destroyed.

The thermal expansion of wood is so small as to

Table 2-3. Deterioration due to four weeks soaking in acids and bases at room temperature measured by the modulus of rupture relative to that for matched water-soaked specimens. Concentrations, 2 to 10 percent (ref. 2-36)

Species	Hydrochloric acid	Sulfuric acid	Nitric acid	Sodium hydroxide	Ammonium hydroxide
Larch	¹ 1.09-0.81	1.11-1.00	1.07-0.95	0.96-0.52	1.06-0.92
Pine	1.08- .84	1.13-0.84	1.07- .93	.96- .51	1.00- .79
Spruce	1.00- .82	1.02- .94	.98- .96	.93- .44	.98- .75
Beech	.96- .55	.97- .87	.83- .71	.59- .31	.67- .43
Oak	.93- .52	.97- .82	.83- .65	.58- .29	.65- .36
Basswood	.88- .48	.87- .81	.76- .57	.55- .21	.61- .33

¹ Values greater than unity due to the fact that acid is more slowly absorbed by the wood than water.

Table 2-4. Loss in breaking load in percent due to soaking in acids at room temperature relative to matched water-soaked specimens (ref. 2-1)

Species	Hydrochloric acid 82 days		Sulfuric acid 82 days		Acetic acid 135 days	
	5 percent concentration	15 percent concentration	10 percent concentration	20 percent concentration	50 percent concentration	80 percent concentration
	Percent	Percent	Percent	Percent	Percent	Percent
Teak	31	50	17	31	43	39
Oak	54	76	48	40	39	26
Pine	18	46	11	16	20	0

be unimportant in ordinary usage, for example, a 1 percent increase in moisture content swells yellow birch as much as does an 80° C. thermal expansion. Only meager data are available on the coefficients of thermal expansion of wood and wood-base materials, and investigators are not in close agreement in their values although there is general agreement that the expansion across the grain is much greater than that along the grain (ref. 2-20).

A comprehensive study of the thermal expansion of wood and wood-base products has been completed recently at the Forest Products Laboratory, and the results have been published (ref. 2-82). In this work the variation of the coefficients of linear thermal expansion with specific gravity was determined on a series of solid, oven-dry specimens of 9 different species of untreated wood. The effects of radial compression, resin-treating, and cross-banding on the values of the coefficients were determined on birch laminates. The values of the coefficients for papreg and hydrolyzed-wood plastic were also determined.

The average coefficients of linear thermal expansion (α) per ° C. of nine species of solid wood (ref. 2-82) from +50 to -50° C. for the average specific gravity of the species are given in the following tabulation:

Species	Average specific gravity ¹	$\alpha_t \times 10^6$	$\alpha_r \times 10^6$	$\alpha_l \times 10^6$
Yellow Birch	0.66	38.3	30.7	3.36
Sugar Maple	.68	35.3	26.8	3.82
Yellow-poplar	.43	29.7	27.8	3.17
Cottonwood	.43	32.6	23.2	2.89
Balsa	² .17	³ 24.1	³ 16.3	-----
Douglas-fir ³	.51	42.7	27.9	3.16
Sitka spruce	.42	32.3	23.8	3.15
White fir	.40	32.6	21.8	3.34
Redwood	.42	35.1	23.6	4.28

¹ Average specific gravity (based on weight and volume when oven dry) taken from tables of properties in U. S. D. A. Tech. Bull. No. 479 (ref 2-57) or for specimens included in this study.

² Specific gravity average of values for two specimens tested.

³ α measured from +50 to 0° C. only.

The coefficients of linear thermal expansion in the tangential (α_t) and radial directions (α_r) may be calculated for specific gravities other than those tabulated by use of the following equation:

$$\alpha = \alpha_o \frac{G}{G_o}$$

(2:1)

where

α =coefficient of linear thermal expansion (α_t or α_r) at specific gravity G

α_c ==coefficient of linear thermal expansion (α_l or α_r) listed

G_o ==specific gravity listed

G ==specific gravity desired

The coefficient of linear thermal expansion parallel to the grain (α_l) is independent of specific gravity, and is unaffected by radial or tangential compression.

The dependency of the coefficient of linear thermal expansion on resin content in a solid piece is expressed by:

$$\alpha_c = \frac{E_w \alpha_w (1 - n_r) + E_r \alpha_r n_r}{E_w (1 - n_r) + E_r n_r} \quad (2:2)$$

where

α_c ==coefficient of linear thermal expansion (α) of the wood-resin system

α_w ==of wood alone

α_r ==of resin alone.

E ==modulus of elasticity (subscripts have the same meaning as for α)

n_r ==fraction of solid cross-section of sample consisting of resin

General formulas were developed at the Forest Products Laboratory that permit calculation of the coefficients of linear thermal expansion of wood laminates in any grain direction of the specimen, from the original and final specific gravities, the percentage by weight of resin and glue present, the percentage of cross-banding, and the slope of grain relative to any three axes of reference. Solution of the formulas, however, should be attempted only after reference to the original publication (ref. 2-82).

2.092. *Thermal conductivity.* The thermal conductivity of wood is dependent on a number of factors of varying degrees of importance. Some of the more significant variables affecting the rate of heat flow in wood are the following: density and moisture content of the wood; direction of heat flow with respect to the grain; kind, quantity, and distribution of extractives or chemical substances; relative density and proportion of springwood and summerwood; defects, like checks, knots, and cross-grain structure.

The Forest Products Laboratory has made careful determinations of the thermal conductivity of wood at various moisture contents. These tests, which covered 32 species, have furnished sufficient data on the relationship between conductivity, specific gravity, and moisture content to make it possible to compute the approximate

thermal conductivity for any wood for which the specific gravity is known and for which the moisture content can be determined or assumed. Such conductivities have many practical applications, such as in estimating the thermal resistance or insulating value of various woods; thermal resistance being the reciprocal or inverse value of conductivity.

It is common engineering practice to express heat conductivity, represented by K , as the amount of heat in British thermal units that will pass in 1 hour through 1 square foot of the material 1 inch thick per degree Fahrenheit temperature difference between the faces.

Although it is not practicable (ref. 2-45) to compute the exact conductivity of wood of given density and moisture content because of the number of variables involved, the following equations permit calculation of conductivity closely enough for practical purposes:

For wood having a moisture content under 40 percent:

$$K = S(1.39 + 0.028M) + 0.165 \quad (2:3)$$

For wood having a moisture content of 40 percent or more:

$$K = S(1.39 + 0.038M) + 0.165 \quad (2:4)$$

Where

K =conductivity

S =specific gravity based on volume at current moisture and weight when oven-dry

M =moisture content

Conductivities of wood aircraft parts will ordinarily be computed by means of equation (2:3) using 15 percent for the moisture content of aircraft to be used in the continental United States and 20 percent for the moisture content of aircraft to be used under tropical conditions (sec. 2.10). The use of species average values of specific gravity may be considered sufficiently correct for most purposes.

Experiments on Douglas-fir plywood with veneer $\frac{1}{8}$ -inch or more in thickness (ref. 2-45) indicate that the thin film of glue between the wood surfaces has no important effect on conductivity, as would be expected, because of the very slight thickness of the glue coating in comparison with the total thickness.

Tests (ref. 2-45) indicate there is a small increase in conductivity with increase in temperature difference but, for temperature conditions normally encountered, the variation in conductivity is not significant.

2.093. *Ignition temperature.* Limited data are available concerning minimum temperatures required to produce charring or ignition of wood. Results obtained by different investigators for ignition temperatures show wide discrepancies. The different values reported may be due to the specific test conditions associated with the methods employed, and also to the different interpretations among investigators as to what constitutes ignition temperature (ref. 2-4). Assuming conditions favorable to the completion of the ignition process, the ignition temperature has been defined (ref. 2-4) as the temperature in the combustible at which the rate of heat developed by the reactions inducing ignition just exceeds the rate at which heat is dissipated by all causes, under the given conditions.

It is thus obvious that, unlike flammable liquids, which have reasonably definite ignition temperatures, the ignition temperature of wood, even if a standard interpretation of the phenomenon were determined upon, would vary widely depending upon the size, density, moisture content, and type, distribution, and quantity of extractives present in the specimen under test, and upon the time and rate of heating, the amount of air available, and the rate of air flow.

The importance of the time factor has been emphasized by the Forest Products Laboratory (ref. 2-49) but no specific tests have been made relating ignition temperatures to long exposures at the lower ranges of elevated temperature. The Underwriters' Laboratories (ref. 2-80) have cited an example of ignition occurring after long-continued exposure (about 15 yrs.) to a temperature of approximately 190° F.

2.094. *Electrical properties.* The resistance that wood offers to the passage of direct current depends primarily upon the moisture content of the wood (ref. 2-72). In the green state the resistivity of wood is relatively low and increases slowly with decrease in moisture until the fiber-saturation point is reached at about 30 percent moisture content, and all free water has been removed. The change of resistivity within the green range is about 50-fold. When wood is dried below the fiber-saturation point, however, its resistivity increases rapidly, about a million-fold from the fiber-satura-

tion point to the oven dry condition. The logarithm of resistivity is approximately inversely proportional to moisture content. At values of moisture content approaching zero the resistivity becomes very great, of the order of 10^{16} ohm-centimeters (ref. 2-83), and dry wood is a very good electrical insulator. In conditions of use, however, wood will not remain dry, but will absorb moisture until it reaches a condition of equilibrium corresponding with the ambient atmosphere. The resistivity of wood at a typical moisture content of 9.3 percent is 10^{10} ohm-centimeters.

When alternating voltage is applied to wood the effects depend upon both moisture content and frequency. At frequencies up to a few hundred cycles per second the behavior of wood is practically the same as for direct currents. At much higher frequencies, from a million cycles per second upward, the electrical properties of wood are essentially its properties when acting as a dielectric material. In this role it is interposed between two metallic plates or sheets to form a condenser. Wood is an imperfect dielectric and, therefore, some of the electrical energy required to charge the condenser will be lost to the wood where it appears as heat.

Losses in the wood depend principally upon its moisture content and the frequency of the applied voltage, and the losses increase with both moisture content, especially above about 10 percent, and frequency. Wood is a very poor insulator or dielectric at high frequencies. The alternating current electrical properties of wood are concerned principally with high-frequency dielectric heating for gluing purposes (ref. 2-5, 2-74).

2.095. *Damping capacity.* Damping capacity may be defined as the ability of a solid to convert mechanical energy of vibration into internal energy. This causes vibrations to die out. Published information on the subject (ref. 2-33) is mainly concerned with metals, and only occasional references are made to wood.

The damping capacity of timber has been investigated by Greenhill (ref. 2-29). His investigations indicate that "if a truly elastic material is subjected to a cycle of stress, the stress-strain curve will be a straight line. If, however, the material undergoes reversible plastic deformation during the cycle, the stress-strain curve will be a hysteresis loop. The area enclosed by this loop represents the amount of energy expended during each complete stress cycle. Specimens subjected to cycles of stress below the fatigue limit can dis-

sipate an unlimited quantity of energy as heat without any damage.

“When a solid is subjected to a periodic force, the damping capacity prevents the amplitude of vibration from becoming infinite when the frequency of the applied force approaches a natural frequency of the solid. Damping capacity is of considerable importance in certain branches of engineering and, consistent with other properties, it is generally agreed that materials of high damping capacity are superior to those of low damping capacity. Take, for example, the wings of aeroplanes. Under certain circumstances these are subject to resonant vibrations, the amplitude of which depends essentially on the damping properties of the materials of construction. The same thing applies with special force to the blades of aeroplane propellers which are liable to vibrate violently at certain critical speeds of rotation. The amplitudes of vibra-

tion are great or small according to the material of which the blades are made. It is stated by experts that the endurance of the blades depends far more on the damping capacity of the material than on its fatigue strength.”

Damping capacity has been expressed numerically in various ways. Greenhill and Kimball (ref. 2-29, 2-33) have used logarithmic decrement (δ), and Kimball has summarized various formulas used by different investigators for determination of this factor. Briefly, if ΔW is the energy dissipated per cycle of vibration, and W the maximum energy of the cycle, the ratio $\Delta W/W$, called the “specific energy loss” or the “damping capacity,” gives a measure of the damping characteristics of the material. The logarithmic decrement δ is equal to one-half the damping capacity, or $\Delta W/2W$.

Values of the logarithmic decrement δ for wood and a few other materials are given in table 2-5.

Table 2-5. Damping capacity—logarithmic decrements (δ) for internal friction in solids

Investigator	Material	Method	Stress-percent of maximum	Logarithmic decrement
Forest Products Laboratory	Wood—Parallel laminated birch	Compression	84	0.002
	do	do	88	.012
	do	do	93	.063
	do	Flexure		.073
Do	Staypak—Yellow birch	Compression	86	.0013
	do	do	89	.0465
	do	do	92	.1435
	do	Flexure		.128
Do	Compreg—Yellow birch, high-impact type	Compression	83	.173
	do	do	89	.232
	do	Flexure		.238
	do	Compression	74	.045
Do	Compreg—Yellow birch, low-impact type	do	87	.157
	do	do	90	.172
	do	Flexure		.108
	do	Compression	89	.0002
Do	Compreg—Maple	do	94	.149
	do	do	93	.218
	do	Flexure		.121
	do	do		.239
Greenhill (Ref. 2-29)	Wood—Sitka spruce	Flexure		.035
	do	Torsion		.0521
Kimball and Lovell (Ref. 2-33).	Wood—Maple	Revolving defl. rods.		.021
	Nickel steel	do		.0023
	Mild steel	do		.0049
	Aluminum	do		.0034
	Nickel	do		.0032
	Celluloid	do		.0450
	Wood	Flexure		.027
A. Gemant	Nickel steel	do		.0017
W. Jackson (Ref. 2-33)	Copper	do		.0032

Data other than those from Greenhill and the Forest Products Laboratory are from a compilation included in reference 2-33.

Greenhill (ref. 2-29) found that the effect of increasing the moisture content of wood is to increase its damping capacity, the relation being practically linear within the range of 8 to 19 percent moisture content examined.

Upon removal of stress, full recovery of strain does not take place in most materials even though the stress imposed may be less than that corresponding to the apparent elastic limit of the material. Tests at the Forest Products Laboratory have shown that, with a compression load applied repeatedly to a specimen of papreg, the set or permanent deformation was increased, but the amount of set added for each load cycle diminished. A straight-line relationship was found when accumulative permanent set was plotted on an arithmetical scale against the number of load cycles plotted on a logarithmic scale. It was found that the higher the load, the greater was the permanent set after the first and all succeeding load cycles.

2.1. Basic Strength and Elastic Properties of Wood

2.10. DESIGN VALUES, TABLES 2-6 AND 2-7. Strength properties of various species for use in calculating the strength of aircraft elements are presented in tables 2-6 and 2-7. Their applicability to the purpose is considered to have been substantiated by experience. The assumptions (see footnotes to tables 2-6 and 2-7) made in deriving the values in tables 2-6 and 2-7 from the results of standard tests (sec. 2.12) have been reexamined in the light of recent data with respect to the distribution of strength values in wood for aircraft construction and the moisture content of airplane parts, together with data relating to "duration of stress" in order to clarify the basis of design (ref. 2-12, 2-13).

The values in table 2-6 are based on a moisture content of 15 percent and are considered applicable for design of structural parts of aircraft that are to be used in the continental United States. Values in table 2-7 are based on a moisture content of 20 percent and should be used for design of structural parts of aircraft to be used under tropical conditions where high relative humidity, approximately 90 percent or over, is prevalent

for long periods of time, or more or less continuously.

When tests of physical properties are made on additional species or on specially selected wood the results may be made comparable to those in tables 2-6 and 2-7 by adjusting them to 15 or 20 percent moisture content, respectively, in accordance with table 2-2, together with the appropriate use of the factors described in the footnotes to tables 2-6 and 2-7.

For notes on acceptable procedures for static tests and the correction of test results, see sections 2.12 and 3.01.

2.100. Supplemental notes.

2.1000. *Compression perpendicular to grain.* Wood does not exhibit a definite ultimate strength in compression perpendicular to the grain, particularly when the load is applied over only a part of the surface, as it is by fittings. Beyond the proportional limit the load continues to increase slowly until the deformation becomes several times as great as at the proportional limit and the crushing is so severe as to damage the wood seriously in other properties. A "probability" factor was applied to average values of stress at proportional limit to take account of variability, and the result was increased by 50 percent to get design values comparable to those for bending, compression parallel to grain, and shear as shown in tables 2-6 and 2-7.

2.1001. *Compression parallel to grain.* Available data indicate that the proportional limit for hardwoods is about 75 percent and for softwoods about 80 percent of the maximum crushing strength. Accordingly, design values for fiber stress at proportional limit were obtained by multiplying maximum crushing-strength values by a factor of 0.75 for hardwoods and 0.80 for softwoods, and adjusting for a difference in the factors for the "rate and duration of load."

2.11. NOTES ON THE USE OF VALUES IN TABLES 2-6 AND 2-7.

2.110. *Relation of design values in tables 2-6 and 2-7 to slope of grain.* The values given in tables 2-6 and 2-7 apply for grain slopes as steep as the following:

- (a) For compression parallel to grain—1 in 12.
- (b) For bending and for tension parallel to grain—1 in 15.

When material is used in which the steepest grain slope is steeper than the above limits, the design values of tables 2-6 and 2-7 must be reduced according to the percentages in table 2-8.

Table 2-6.—Strength values of various woods, based on 15 percent moisture content,¹ to be used in design of aircraft for use in the continental United States (see sections 2.10 and 2.11 for explanations relative to the basis for, and use of, the values in this table)

Species of wood: common and botanical names	Specific gravity based on volume and weight when oven dry		Weight at 15 percent moisture content	Shrinkage from green to oven-dry condition based on dimensions when green		Static bending				Compression parallel to grain		Compression perpendicular to grain ¹ (<i>F</i> _{c⊥})	Shearing strength parallel to grain ¹ (<i>F</i> _{sv})	Hardness: load required to indent 0.444-inch ball to one-half its diameter	Tension	
	Average ²	Min-imum per-mitted ³		Radial	Tangen-tial	Fiber stress at propor-tional limit ⁴ (<i>F</i> _b)	Modulus of rup-ture ⁴ (<i>F</i> _b)	Modulus of elas-ticity ⁴ (<i>E</i> _t)	Work to maxi-mum load	Fiber stress at propor-tional limit ⁴ (<i>F</i> _{cp})	Maxi-mum crushing strength ⁴ (<i>F</i> _{cu})				Strength parallel to grain ⁵ (<i>F</i> _{tr})	Strength perpen-dicular to grain ⁵ (<i>F</i> _{tr⊥})
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)		
HARDWOODS (BROAD-LEAVED SPECIES)																
Ash, black (<i>Fraxinus nigra</i>)	0.53	0.48	35	Percent	Percent	<i>P. s. i.</i>	<i>P. s. i.</i>	<i>P. s. i.</i>	<i>In.-lb. per cu. ft.</i>	<i>P. s. i.</i>	<i>P. s. i.</i>	<i>P. s. i.</i>	<i>P. s. i.</i>	<i>Lb.</i>	Tension strength parallel to grain taken as equal to modulus of rupture. See section 2.11.	<i>P. s. i.</i>
Ash, commercial white (<i>Fraxinus spp.</i>) ¹⁰	.02	.56	41	5.0	7.8	5,200	10,600	1,310	14.2	3,220	4,600	910	1,190	750		
Basswood, American (<i>Tilia glabra</i>)	.40	.36	25	4.5	7.2	7,200	13,200	1,400	14.9	4,270	6,100	1,080	1,500	1,180		
Beech, American (<i>Fagus grandifolia</i>)	.67	.60	43	6.6	9.3	4,500	7,700	1,220	6.6	2,680	3,800	450	770	360		
Birch, Alaska (<i>Betula neolaskana</i>)	.59	.53	38	5.1	11.0	6,000	13,200	1,480	14.2	4,180	6,000	1,300	1,600	1,170		
Birch, paper (<i>Betula papyrifera</i>)	.60	.54	38	6.5	9.9	6,000	11,800	1,580	13.2	4,070	5,800	850	1,120	750		
Birch (<i>Betula spp.</i>) ¹¹	.08	.58	44	6.3	8.6	5,300	10,600	1,340	16.1	3,130	4,500	740	980	810		
Cherry, black (<i>Prunus serotina</i>)	.53	.48	36	6.8	8.8	7,600	15,100	1,850	18.6	4,500	6,500	1,380	1,630	1,230		
Cottonwood, Eastern (<i>Populus deltoides</i>)	.43	.39	28	3.7	7.1	6,900	11,300	1,310	11.7	4,090	5,800	880	1,300	870		
Elm, American (<i>Ulmus americana</i>)	.55	.50	35	3.9	9.2	4,500	7,700	1,150	7.3	2,770	4,000	400	700	400		
Elm, rock (<i>Ulmus thomasi</i>)	.66	.60	43	4.2	9.5	6,000	10,600	1,160	12.7	3,220	4,600	880	1,200	770		
Hickory (true hickories) (<i>Hicoria spp.</i>) ¹²	.80	.71	50	4.8	8.1	6,500	13,500	1,310	19.3	4,130	5,900	1,550	1,530	1,210		
Khaya ("African mahogany") (<i>Khaya spp.</i>) ¹³	.48	.42	32	7.4	11.4	8,700	17,100	1,780	26.1	5,080	7,200	2,250	1,600			
Locust, black (<i>Robinia pseudoacacia</i>)	.71	.64	49	4.1	5.8	5,800	9,800	1,260	7.9	3,480	5,000	1,320	1,110	720		
Magnolia, Southern (<i>Magnolia grandiflora</i>)	.53	.48	35	4.4	6.9	10,900	18,100	1,810	17.6	6,300	9,000	2,450	2,010	1,670		
Mahogany (<i>Swietenia spp.</i>) ¹⁴	.52	.46	35	5.4	6.6	5,500	10,100	1,200	13.4	3,140	4,500	1,100	1,230	940		
Maple, silver (<i>Acer saccharinum</i>)	.51	.46	33	3.5	4.8	6,800	11,700	1,370	7.4	4,330	6,200	1,370	1,090	710		
Maple, sugar (<i>Acer saccharum</i>)	.68	.60	43	3.0	7.2	4,900	8,100	990	8.9	2,970	4,200	930	1,200	670		
Oak, commercial white and red (<i>Quercus spp.</i>) ¹⁵	.69	.62	44	4.9	9.5	7,600	14,100	1,590	15.7	4,540	6,500	1,790	1,830	1,310		
Pecan (<i>Hicoria pecan</i>)	.60	.62	46	4.8	9.4	6,600	12,300	1,460	13.2	3,990	5,700	1,450	1,470	1,230		
Sweetgum (<i>Liquidambar styraciflua</i>) ¹⁶	.54	.48	35	4.9	8.9	7,400	12,800	1,480	14.0	4,540	6,500	2,120	1,690	1,680		
Sycamore, American (<i>Platanus occidentalis</i>)	.51	.49	35	5.2	9.9	6,200	11,200	1,340	10.8	3,380	4,800	770	1,170	610		
Tupelo, water (<i>Nyssa aquatica</i>)	.52	.47	35	5.1	7.6	5,100	9,100	1,200	8.2	3,160	4,500	890	1,180	730		
Walnut, black (<i>Juglans nigra</i>)	.56	.52	38	4.2	7.6	5,900	9,100	1,090	7.2	3,520	5,000	1,120	1,310	830		
Yellow-poplar (<i>Liriodendron tulipifera</i>) ¹⁷	.44	.38	29	5.2	7.1	8,300	13,300	1,460	11.5	4,510	6,400	1,270	1,180	980		
				4.0	7.1	4,600	8,600	1,250	6.4	2,890	4,100	620	930	120		250

See footnotes at end of table.

Table 2-6.—Strength values of various woods, based on 15 percent moisture content,¹ to be used in design of aircraft for use in the continental United States (see sections 2.10 and 2.11 for explanations relative to the basis for, and use of, the values in this table)—Continued

Species of wood; common and botanical names	Specific gravity based on volume and weight when oven dry		Weight at 15 percent moisture content	Shrinkage from green to oven-dry condition based on dimensions when green		Static bending				Compression parallel to grain		Compression perpendicular to grain (F_{\perp})	Shearing strength parallel to grain (F_{\parallel})	Hardness, side load required to embed 0.444-inch ball to one-half its diameter	Tension	
	Average ²	Minimum permitted ³		Radial	Tangential	Fiber stress at proportional limit (F_b)	Modulus of rupture (F_{bu})	Modulus of elasticity (E_s)	Work to maximum load	Fiber stress at proportional limit (F_{cp})	Maximum crushing strength (F_{cw})				(16)	(17)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
SOFTWOODS (CONIFERS)																
Baldcypress (<i>Taxodium distichum</i>)	0.48	0.43	Lb. per cu. ft. 32	Percent 3.8	Percent 6.2	P. s. i. 5,900	P. s. i. 9,600	1,000 p. s. i.	In.-lb. per cu. ft. 7.7	P. s. i. 4,020	P. s. i. 5,400	P. s. i. 940	P. s. i. 840	Lb. 480	Tension strength parallel to grain taken as equal to modulus of rupture. See section 2.11.	
Cedar, Alaska (<i>Chamaecyparis nootkensis</i>)	.46	.41	31	2.8	6.0	5,700	9,900	1,220	10.1	3,840	5,100	810	930	540		
Douglas-fir (normal) (<i>Pseudotsuga taxifolia</i>) ¹²	.50	.45	33	5.0	7.8	5,900	10,900	1,430	8.1	4,220	5,600	1,020	950	610		
Douglas-fir (light) ^{12 16}	.43	.38	28			4,900	9,000	1,210		3,300	4,500	770				
Fir, California red (<i>Abies magnifica</i>)	.42	.38	28	4.5	7.9	5,100	9,400	1,320	8.2	3,480	4,700	680	890	400		
Fir, noble (<i>Abies nobilis</i>) ¹²	.42	.36	28	4.5	8.3	5,600	9,800	1,470	8.0	3,580	4,800	700	880	380		
Fir, Pacific (<i>Abies amabilis</i>)	.42	.38	27	4.5	10.0	5,000	8,400	1,320	8.3	3,370	4,500	530	830	400		
Fir, white (<i>Abies concolor</i>)	.40	.36	26	3.2	7.0	5,300	8,400	1,170	6.3	3,200	4,400	650	780	410		
Hemlock, Western (<i>Tsuga heterophylla</i>) ¹²	.45	.40	30	4.3	7.9	6,200	11,000	1,510	7.3	4,080	5,500	730	860	540		
Incense-cedar, California (<i>Libocedrus decurrens</i>)	.36	.32	25	3.3	5.2	5,000	7,600	900	5.6	3,350	4,500	700	760	450		
Larch, Western (<i>Larix occidentalis</i>)	.59	.53	37	4.2	8.1	6,700	11,000	1,480	7.8	4,780	6,400	1,150	1,110	680		
Pine, Eastern white (<i>Pinus strobus</i>)	.37	.34	25	2.1	6.1	4,600	7,600	1,060	6.3	2,960	4,000	520	740	350		
Pine, ponderosa (<i>Pinus ponderosa</i>)	.42	.33	23	3.9	6.3	5,000	8,100	1,070	6.2	3,160	4,200	750	900	410		
Pine, red (<i>Pinus resinosa</i>)	.51	.46	33	4.6	7.2	7,000	10,500	1,530	8.7	4,320	5,800	820	970	510		
Pine, sugar (<i>Pinus lambertiana</i>)	.38	.34	26	2.9	5.6	4,700	7,300	1,020	5.5	2,970	4,000	630	830	360		
Pine, Western white (<i>Pinus monterea</i>)	.42	.38	28	4.1	7.4	5,000	8,300	1,280	7.7	3,400	4,600	560	700	360		
Redcedar, Western (<i>Thuja plicata</i>)	.34	.31	23	2.4	5.0	4,400	7,100	960	5.6	3,160	4,200	640	720	330		
Spruce, Sitka (<i>Picea sitchensis</i>) ¹²	.41	.36	28	4.2	7.5	5,300	9,400	1,350	8.7	3,530	4,700	740	900	470		
Spruce, red (<i>Picea rubra</i>)	.41	.36	28	3.8	7.8	5,600	9,300	1,320	8.1	3,670	4,900	630	800	460		
Spruce, white (<i>Picea glauca</i>)	.45	.36	29	4.7	8.2	5,100	8,700	1,150	7.2	3,310	4,400	500	850	440		
White-cedar, Northern (<i>Thuja occidentalis</i>)	.32	.29	22	2.1	4.7	3,900	6,000	690	5.0	2,430	3,300	430	700	280		
White-cedar, Port Orford (<i>Chamaecyparis lawsoniana</i>) ^{12 16}	.43	.40	29	4.6	6.9	6,700	10,200	1,430	8.6	4,110	5,500	810	790	510		

¹ A moisture content value of 15 percent has been used in design of wood aircraft for use in the continental United States for many years. This value was substantiated as a normally expected service condition by reference 2-12. For design of aircraft to be used under tropical conditions see table 2-7 and section 2.10.

² Values in column 2 for birch, klaysa, mahogany, sweetgum, yellow-poplar, Douglas-fir, noble fir, Western hemlock, Sitka spruce, and Port Orford white cedar are averages for material having a specific gravity equal to or greater than the minimum permitted for each species, as listed in column 3. For species other than those listed above, values in column 2 are general species averages, without excluding material below the minimum permitted.

³ Values in column 3 for minimum permitted specific gravity are those listed in table 2-3 of ANO-18, June, 1944, corresponding to applicable Army-Navy-Aeronautical specifications as of that date. The continued applicability of these values is considered as being substantiated by experience.

⁴ The values in columns 7, 8, 9, 12, 13, and 14 are species mean values adjusted to 15 percent moisture content (see footnote 1) and multiplied by two factors: (1) A "probability" factor to allow for variability of wood, and (2) a "rate and duration of load" factor to allow for the effect of estimated loading conditions for aircraft in service. The "probability" factor is based on composite cumulative frequency curves (figs. 1 and 2, ref. 2-13). Factors were determined for each property at an "inclusion" limit of 25 percent; that is, they represent strength values, expressed as percentages of the respective means, below which not more than 25 percent of aircraft-grade material would be expected to fail.

The "rate and duration of load" factors for ultimate are those applicable to a loading time of 1 second and a duration of 1.5 seconds, while those for proportional limits relate to a loading time of 3 seconds and a duration of 15 seconds (data from ref. 2-3). Values for modulus of elasticity in static bending, for compression perpendicular to grain, and for shear are assumed to be unaffected by rate and duration of load. Factors used for the several properties are:

Test	Property	"Probability" Factor	"Rate and Duration of Load" Factor
Static bending	Fiber stress at proportional limit.	0.89	1.05
Static bending	Modulus of rupture.	.92	1.10
Static bending	Modulus of elasticity.	.91	1.00
Compression parallel to grain	Fiber stress at proportional limit.	(See footnote 6).	1.00
Compression parallel to grain	Maximum crushing strength.	.91	1.07
Compression perpendicular to grain	Fiber stress at proportional limit.	.81	1.00
Shear parallel to grain	Maximum shearing strength.	.88	1.00

⁵ The values given in column 9 are based on the average apparent modulus of elasticity (E_a) as obtained by substituting results from tests of 2- by 2-inch beams on a 28-inch span or 1- by 1-inch beams on a 14-inch span with load at the center in the formula $E_a = PL^3/48\delta$. The use of these values of E_a in the usual formulas will give the deflection of beams of ordinary length with but small error. For exactness in the computation of deflections of I and box beams, particularly for short spans, the

formula that takes into account shear deformations (see National Advisory Committee for Aeronautics Report No. 180, ref. 2-59) should be used. This formula involves E_s , the true modulus of elasticity in bending, and G , the modulus of rigidity in shear. Values of E_s may be obtained by adding 10 percent to the values of E_a as given in the table. If the I or box beam has the grain of the web parallel to the axis of the beam, or parallel and perpendicular thereto as in some plywood webs, the value of G may be taken as $E_s/16$ or $E_s/14.5$. If the web is of plywood with the grain at 45° to the axis of the beam G may be taken as $E_s/5$ or $E_s/4.5$.

⁶ Design values for fiber stress at proportional limit in compression parallel to grain were obtained by multiplying the values of maximum crushing strength in the next column by 0.75 for hardwoods and 0.80 for conifers, and by the ratio of the "rate and duration of load" factors, namely, 1.00/1.07.

⁷ Wood does not exhibit a definite ultimate strength in compression perpendicular to grain, particularly when the load is applied over only a part of the surface, as at fittings. Beyond the proportional limit, the load continues to increase slowly until the deformation becomes several times as great as at the proportional limit (see fig. 2-4) and the crushing is so severe as to damage the wood seriously in other properties. In calculating values in column 13, a "rate and duration of load" factor of 1.00 was assumed, and the average proportional limit stress, modified by the appropriate "probability" factor (see footnote 4) was increased by 50 percent to get design values.

⁸ Values in this column are for use in computing resistance to longitudinal shear. They were obtained by multiplying average values by a "probability" factor (see footnote 4) to allow for variability.

Tests have shown that because of the favorable influence upon the distribution of stresses resulting from limiting shearing deformations the maximum strength-weight ratio and minimum variability in strength are attained when I and box beams are so proportioned that the ultimate shearing strength is not developed and failure by shear does not occur.

⁹ Values in column 17 are one-half the average values at 15 percent moisture content.

¹⁰ Includes white ash (*F. americana*), green ash (*F. pennsylvanica lanceolata*), and blue ash (*F. quadrangulata*).

¹¹ Includes sweet birch (*B. lenta*) and yellow birch (*B. lutea*).

¹² Values for these species in columns 2, 4, 7, 8, 9, 11, 12, and 13 are based on material conforming to aircraft specifications for minimum permitted specific gravity and rings per inch (see footnote 2 and reference 2-13). Values in columns 5, 6, 10, 14, 15, and 17, and all values for other species, are based on general averages for the species without excluding material below the minimum permitted specific gravity or with less than the required number of rings per inch.

¹³ Includes shagbark hickory (*H. latifolia*), mockernut hickory (*H. alba*), pignut hickory (*H. glabra*), and shagbark hickory (*H. ovata*).

¹⁴ Includes only material from Central America.

¹⁵ Includes white oak (*Q. alba*), bur oak (*Q. macrocarpa*), swamp chestnut oak (*Q. prinus*), post oak (*Q. stellata*), Northern red oak (*Q. borealis*), Southern red oak (*Q. rubra*), laurel oak (*Q. laurifolia*), water oak (*Q. nigra*), swamp red oak (*Q. pagodaefolia*), willow oak (*Q. phellos*), and yellow oak (*Q. velutina*).

¹⁶ These species will be found in the Army-Navy-Aeronautical specifications under the following names: White-cedar, Port Orford—(AN-C-72) cedar, aircraft Port Orford; Douglas-fir—(AN-F-7) fir, aircraft Douglas; yellow-poplar (AN-P-17) poplar, aircraft yellow.

Table 2-7. Strength values of various woods based on 20 percent moisture content¹ to be used in design of aircraft for use in tropical regions having continued high relative humidity (approximately 90 percent or over) (see sections 2-10 and 2-11 for explanations relative to the basis for, and use of, the values in this table)

Species of wood; common and botanical names	Specific gravity based on volume and weight when oven dry		Weight at 20 percent moisture content	Shrinkage from green to oven-dry condition based on dimensions when green		Static bending				Compression parallel to grain		Compression perpendicular to grain ⁴ (<i>F</i> _⊥)	Shearing strength parallel to grain ⁴ (<i>F</i> _∥)	Hardness, side load required to imprint 0.444-inch ball to one-half its diameter	Tension	
	Average ²	Minimum permitted ³		Radial	Tangential	Fiber stress at proportional limit ⁵ (<i>F</i> _{br})	Modulus of rupture (<i>F</i> _{br})	Modulus of elasticity (<i>E</i> _L)	Work to maximum load	Fiber stress at proportional limit ⁵ (<i>F</i> _{cr})	Maximum crushing strength (<i>F</i> _{ca})				Strength parallel to grain (<i>F</i> _∥)	Strength perpendicular to grain (<i>F</i> _⊥)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
HARDWOODS (BROAD-LEAVED SPECIES)																
Ash, black (<i>Fraxinus nigra</i>)	0.53	0.48	35	Percent 5.0	Percent 7.8	P. s. i. 3,400	P. s. i. 7,800	1,000 P. s. i.	13.0	P. s. i. 2,160	P. s. i. 3,100	P. s. i. 680	P. s. i. 930	Lb. 610		P. s. i. 276
Ash, commercial; white (<i>Fraxinus spp.</i>) ¹⁰	.62	.56	41	4.5	7.2	5,900	11,100	1,310	14.6	3,390	4,800	1,330	1,380	1,050		328
Basswood, American (<i>Tilia glabra</i>)	.40	.36	25	6.6	9.3	3,200	6,100	1,060	5.8	1,960	2,800	330	620	200		152
Beech, American (<i>Fagus grandifolia</i>)	.67	.60	44	5.1	11.0	5,100	10,500	1,350	12.9	3,090	4,400	1,010	1,320	980		402
Birch, Alaska (<i>Betula neolascana</i>)	.59	.53	38	6.5	9.9	4,500	9,000	1,380	12.3	2,800	4,000	650	930	640		146
Birch, paper (<i>Betula papyrifera</i>)	.60	.54	38	6.3	8.6	3,700	8,100	1,180	16.1	2,160	3,100	530	840	660		
Birch (<i>Betula spp.</i>) ¹¹	.68	.58	44	6.8	8.8	5,600	11,900	1,660	17.3	3,330	4,800	1,020	1,320	1,040		294
Cherry, black (<i>Prunus serotina</i>)	.53	.48	36	3.7	7.1	5,100	9,400	1,240	12.3	3,060	4,400	670	1,140	750		284
Cottonwood, Eastern (<i>Populus deltoides</i>)	.43	.39	29	3.9	9.2	3,400	6,300	1,020	7.3	2,010	2,900	370	670	370		220
Elm, American (<i>Ulmus americana</i>)	.55	.50	36	4.2	9.5	4,600	8,600	1,080	12.2	2,470	3,500	670	1,020	680		308
Elm, rock (<i>Ulmus thomasi</i>)	.66	.60	44	4.8	8.1	5,200	11,200	1,180	19.6	3,180	4,500	1,160	1,200	1,060		
Hickory (true hickories) (<i>Hicoria spp.</i>) ¹³	.80	.71	50	7.4	11.4	6,800	13,600	1,550	27.1	3,820	5,500	1,640	1,380	620		218
Klavya ("African mahogany") (<i>Khaya spp.</i>) ¹²	.48	.42	33	4.1	5.8	5,000	8,700	1,160	8.2	2,970	4,200	1,140	1,690			362
Locust, black (<i>Robinia pseudo-acacia</i>)	.71	.64	50	4.4	6.9	9,300	15,700	1,740	16.3	5,330	7,600	2,020	1,740	1,610		
Magnolia, Southern (<i>Magnolia grandiflora</i>)	.53	.48	36	5.4	6.6	4,200	8,200	1,090	14.5	2,330	3,300	850	1,050	820		326
Mahogany (<i>Swietenia spp.</i>) ^{12 14}	.52	.46	36	3.5	4.8	6,000	10,900	1,320	8.6	3,820	5,500	1,130	1,150	700		173
Maple, silver (<i>Acer saccharinum</i>)	.51	.46	34	3.0	7.2	3,700	6,800	910	10.0	2,180	3,100	700	1,040	630		272
Maple, sugar (<i>Acer saccharum</i>)	.68	.60	44	4.9	9.5	5,900	11,400	1,490	14.3	3,440	4,900	1,270	1,510	1,110		
Oak, commercial white and red (<i>Quercus spp.</i>) ¹⁵	.69	.62	45	4.8	9.4	5,300	10,000	1,290	12.1	3,000	4,300	1,170	1,240	1,120		372
Pecan (<i>Hicoria pecan</i>)	.69	.62	46	4.9	8.9	5,900	11,100	1,340	14.3	3,420	4,900	1,520	1,470	1,400		
Sweetgum (<i>Liquidambar styraciflua</i>) ¹²	.54	.48	35	5.2	9.9	4,500	8,900	1,210	10.0	2,510	3,600	600	900	570		206
Sycamore, American (<i>Platanus occidentalis</i>)	.54	.49	35	5.1	7.6	3,800	7,600	1,040	7.8	2,450	3,500	680	1,000	690		330
Tupelo, water (<i>Nyssa aquatica</i>)	.52	.47	35	4.2	7.6	4,700	8,100	1,010	7.8	2,790	4,000	850	1,160	760		316
Walnut, black (<i>Juglans nigra</i>)	.56	.52	39	5.2	7.1	6,300	11,100	1,360	13.2	3,550	5,100	930	1,120	930		304
Yellow-poplar (<i>Liriodendron tulipifera</i>) ^{12 16}	.44	.38	30	4.0	7.1	3,600	6,900	1,100	5.9	2,070	3,000	490	790	370		237

equal to modulus of rupture. See sec. 2-111.

SOFTWOODS (CONIFERS)

Balkeypress (<i>Taxodium distichum</i>)	.48	.43	33	3.8	6.2	4,700	7,900	1,150	7.1	3,170	4,200	740	770	430	146
Cedar, Alaska (<i>Chamaecyparis nootkatensis</i>)	.46	.41	32	2.8	6.0	4,400	7,900	1,110	9.6	2,840	3,800	640	820	490	109
Douglas-fir (normal) (<i>Pseudotsuga taxifolia</i>) ^{1a}	.50	.45	34	5.0	7.8	4,700	9,100	1,350	7.4	3,230	4,300	800	870	530	130
Douglas-fir (light) ^{1a}	.43	.38	29			4,000	7,500	1,110		2,590	3,500	600			
Fir, California red (<i>Abies magnifica</i>)	.42	.38	20	4.5	7.9	3,900	7,400	1,190	7.1	2,610	3,500	530	780	400	100
Fir, noble (<i>Abies nobilis</i>) ^{1a}	.42	.36	28	4.5	8.3	4,300	7,800	1,340	6.8	2,670	3,400	530	700	320	114
Fir, Pacific silver (<i>Abies amabilis</i>)	.42	.38	27	4.5	10.0	4,000	6,800	1,220	6.9	2,490	3,300	420	690	350	
Fir, white (<i>Abies concolor</i>)	.40	.36	27	3.2	7.0	4,300	6,800	1,040	5.6	2,480	3,300	530	710	370	140
Hemlock, Western (<i>Tsuga heterophylla</i>) ^{1a}	.45	.40	30	4.3	7.9	4,900	9,400	1,410	7.1	3,190	4,300	610	760	490	154
Incense-cedar, California (<i>Libocedrus decurrens</i>)	.36	.32	26	3.3	5.2	4,200	6,900	820	6.0	2,720	3,600	650	750	410	140
Larch, Western (<i>Larix occidentalis</i>)	.50	.53	38	4.2	8.1	5,000	9,500	1,360	7.5	3,810	5,100	930	970	580	131
Pine, Eastern white (<i>Pinus strobus</i>)	.37	.34	26	2.1	6.1	3,500	6,000	970	5.7	2,240	3,000	300	680	320	135
Pine, ponderosa (<i>Pinus ponderosa</i>)	.42	.38	29	3.9	6.3	3,700	6,300	960	5.6	2,280	3,000	550	720	350	162
Pine, red (<i>Pinus resinosa</i>)	.51	.46	34	4.6	7.2	4,800	8,200	1,370	7.0	3,000	4,000	580	860	410	131
Pine, sugar (<i>Pinus lambertiana</i>)	.38	.34	27	2.9	5.6	3,800	6,000	920	5.4	2,290	3,100	510	600	330	148
Pine, Western white (<i>Pinus monticola</i>)	.42	.38	28	4.1	7.4	3,900	6,500	1,150	6.1	2,490	3,300	440	620	310	
Redcedar, Western (<i>Thuja plicata</i>)	.31	.31	24	2.4	5.0	3,500	6,000	880	5.3	2,450	3,300	500	670	290	114
Spruce, Sitka (<i>Picea sitchensis</i>) ^{1a}	.41	.36	28	4.2	7.5	4,200	7,700	1,270	7.6	2,760	3,700	580	860	420	151
Spruce, red (<i>Picea rubra</i>)	.41	.36	29	3.8	7.8	4,400	7,700	1,210	7.5	2,810	3,800	530	790	410	135
Spruce, white (<i>Picea glauca</i>)	.45	.36	29	4.7	8.2	3,900	6,000	1,040	6.5	2,420	3,200	440	710	370	128
White-cedar, Northern (<i>Thuja occidentalis</i>)	.32	.29	22	2.1	4.7	3,000	5,000	630	5.4	1,800	2,400	380	610	250	120
White-cedar, Port Orford (<i>Chamaecyparis lawsoniana</i>) ^{1a}	.43	.40	30	4.6	6.9	5,100	7,900	1,320	7.9	3,040	4,100	580	710	440	116

Tension strength parallel to grain taken as

See footnotes on page 26.

FOOTNOTES FOR TABLE 2-7.

¹ A moisture content value of 20 percent has been used in determining the values in table 2-7. This moisture content represents an equilibrium with high relative humidity, approximately 90 percent or over, corresponding to conditions in tropical areas where high humidity is prevalent for long periods of time, or more or less continuously. For design of aircraft to be used within the continental United States see table 2-6 and section 2.10.

² Values in column 2 for birch, Khaya, mahogany, sweetgum, yellow-poplar, Douglas-fir, noble fir, Western hemlock, Sitka spruce, and Port Orford white-cedar are averages for material having a specific gravity equal to or greater than the minimum permitted for each species, as listed in column 3. For species other than those listed above, values in column 2 are general species averages, without excluding material below the minimum permitted.

³ Values in column 3 for minimum permitted specific gravity are those listed in table 2-3 of ANO-18, June, 1944, corresponding to applicable Army-Navy-Aeronautical specifications as of that date. The continued applicability of these values is considered as being substantiated by experience.

⁴ The values in columns 7, 8, 9, 12, 13, and 14 are species mean values adjusted to 20 percent moisture content (see footnote 1) and multiplied by two factors: (1) A "probability" factor to allow for variability of wood, and (2) a "rate and duration of load" factor to allow for the effect of estimated loading conditions for aircraft in service. The "probability" factor is based on composite cumulative frequency curves (figs. 1 and 2, ref. 2-13). Factors were determined for each property at an "inclusion" limit of 25 percent; that is, they represent strength values, expressed as percentages of the respective means, below which not more than 25 percent of aircraft-grade material would be expected to fail.

The "rate and duration of load" factors for ultimates are those applicable to a loading time of 1 second and a duration of 1.5 seconds, while those for proportional limits relate to a loading time of 3 seconds and a duration of 15 seconds (data from ref. 2-3). Values for modulus of elasticity in static bending, for compression perpendicular to grain, and for shear are assumed to be unaffected by rate and duration of load. Factors used for the several properties are:

Test	Property	"Probability" Factor	"Rate and Duration of Load" Factor
Static bending	Fiber stress at proportional limit.	0.89	1.05
Static bending	Modulus of rupture	.92	1.10
Static bending	Modulus of elasticity	.91	1.00
Compression parallel to grain	Fiber stress at proportional limit.	(See footnote 6)	1.00
Compression parallel to grain	Maximum crushing strength.	.91	1.07
Compression perpendicular to grain.	Fiber stress at proportional limit.	.81	1.00
Shear parallel to grain	Maximum shearing strength	.88	1.00

⁵ The values given in column 9 are based on the average apparent modulus of elasticity (E_a) as obtained by substituting results from tests of 2- by 2-inch beams on a 28-inch span or 1- by 1-inch beams on a 14-inch span with load at the center in the formula $E_a = PL^3/48\delta I$. The use of these values of E_a in the usual formulas will give the deflection of beams of ordinary length with but small error.

For exactness in the computation of deflections of I and box beams, particularly for short spans, the formula that takes into account shear deformations (see National Advisory Committee for Aeronautics Report No. 180, ref. 2-50) should be used. This formula involves E_s , the true modulus of elasticity in bending, and G , the modulus of rigidity in shear. Values of E_s may be obtained by adding 10 percent to the values of E_x as given in the table. If the I or box beam has the grain of the web parallel to the axis of the beam, or parallel and perpendicular thereto as in some plywood webs, the value of G may be taken as $E_x/16$ or $E_y/14.5$. If the web is of plywood with the grain at 45° to the axis of the beam G may be taken as $E_x/5$ or $E_y/4.5$.

⁶ Design values for fiber stress at proportional limit in compression parallel to grain were obtained by multiplying the values of maximum crushing strength in the next column by 0.75 for hardwoods and 0.80 for conifers, and by the ratio of the "rate and duration of load" factors, namely, 1.00/1.07.

⁷ Wood does not exhibit a definite ultimate strength in compression perpendicular to grain, particularly when the load is applied over only a part of the surface, as at fittings. Beyond the proportional limit, the load continues to increase slowly until the deformation becomes several times as great as at the proportional limit (see figure 2-4) and the crushing is so severe as to damage the wood seriously in other properties. In calculating values in column 13, a "rate and duration of load" factor of 1.00 was assumed, and the average proportional limit stress, modified by the appropriate "probability" factor (see footnote 4) was increased by 50 percent to get design values.

⁸ Values in this column are for use in computing resistance to longitudinal shear. They were obtained by multiplying average values by a "probability" factor (see footnote 4) to allow for variability. Tests have shown that because of the favorable influence upon the distribution of stresses resulting from limiting shearing deformations the maximum strength-weight ratio and minimum variability in strength are attained when I and box beams are so proportioned that the ultimate shearing strength is not developed and failure by shear does not occur.

⁹ Values in column 17 are one-half the average values at 20 percent moisture content.

¹⁰ Includes white ash (*F. americana*), green ash (*F. pennsylvanica lanceolata*), and blue ash (*F. quadrangulata*).

¹¹ Includes sweet birch (*B. lenta*) and yellow birch (*B. lutea*).

¹² Values for these species in columns 2, 4, 7, 8, 9, 11, 12, and 13 are based on material conforming to aircraft specifications for minimum permitted specific gravity and rings per inch (see footnote 2 and ref. 2-13). Values in columns 5, 6, 10, 11, 15, and 17, and all values for other species, are based on general averages for the species without excluding material below the minimum permitted specific gravity or with less than the required number of rings per inch.

¹³ Includes shagbark hickory (*H. incinus*), mockernut hickory (*H. alba*), pignut hickory (*H. glabra*), and shagbark hickory (*H. ovata*).

¹⁴ Includes only material from Central America.

¹⁵ Includes white oak (*Q. alba*), bur oak (*Q. macrocarpa*), swamp chestnut oak (*Q. prinus*), post oak (*Q. stellata*), Northern red oak (*Q. borealis*), Southern red oak (*Q. rubra*), laurel oak (*Q. laurifolia*), water oak (*Q. nigra*), swamp red oak (*Q. pagodifolia*), willow oak (*Q. phellos*), and yellow oak (*Q. reticulata*).

¹⁶ These species will be found in the Army-Navy-Aeronautical specifications under the following names: White-cedar, Port Orford—(AN-C-72) cedar, aircraft Port Orford; Douglas-fir (AN-F-7) fir, aircraft Douglas; yellow-poplar (AN-F-17) poplar, aircraft yellow.

2.111. *Tension parallel to grain.* Relatively few data are available on the tensile strength of various species parallel to grain. In the absence of sufficient tensile-test data upon which to base tension design values, the values used in design for modulus of rupture are used also for tension. While it is recognized that this is somewhat conservative, the pronounced effect of stress concentration, slope of grain (table 2-8) and other factors upon tensile strength makes the use of conservative values desirable.

Pending further investigation of the effects of stress concentration at bolt holes, it is recommended that the stress in the area remaining to resist tension at the critical section through a bolt hole not exceed two-thirds the modulus of rupture in static bending when cross-banded reinforcing plates are used; otherwise one-half the modulus of rupture shall not be exceeded.

2.112. *Tension perpendicular to grain.* Values of strength of various species in tension perpendicular to grain have been included for use as a guide in estimating the adequacy of glued joints subjected to such stresses. For example, the joints between the upper wing skin and wing framework are subjected to tensile stresses perpendicular to the grain by reason of the lift forces exerted on the upper skin surface.

Caution must be exercised in the use of these values, since little experience is available to serve as a guide in relating these design values to the average property. Considering the variability of this property, however, the possible discontinuity or lack of uniformity of glue joints, and the probable concentration of stress along the edges of such joints, the average test values for each species have been multiplied by a factor of 0.5 to obtain the values given in tables 2-6 and 2-7.

Table 2-8. Reduction in wood strength for various grain slopes

Maximum slope of grain in the member	Corresponding design value, percent of value in table 2-6				
	Static bending			Compression parallel to grain	Tension parallel to grain
	Fiber stress at proportional limit	Modulus of rupture	Modulus of elasticity	Maximum crushing strength	Modulus of rupture
1 in 15----	100	100	100	-----	100
1 in 12----	98	88	97	100	85
1 in 10----	87	78	91	98	75
1 in 8----	78	67	84	94	60

2.12. STANDARD TEST PROCEDURES.

2.120. *Static bending.* In the static-bending test, the resistance of a beam to slowly applied loads is measured. The beam is 2 by 2 inches in cross section and 30 inches long and is supported on roller bearings which rest on knife edges 28 inches apart. Load is applied at the center of the length through a hard maple block 3¹/₈ inches wide, having a compound curvature. The curvature has a radius of 3 inches over the central 2¹/₈ inches of arc, and is joined by an arc of 2-inch radius on each side. The standard placement is with the annual rings of the specimen horizontal and the loading block bearing on the side of the piece nearest the pith. A constant rate of deflection (0.1 inch per minute) is maintained until the specimen fails. Load and deflection are read simultaneously at suitable intervals.

Figure 2-4 (a) shows a static-bending test set-up, and typical load-deflection curves for Sitka spruce and yellow birch.

Data on a number of properties are obtained from this test. These are discussed as follows:

2.1200. *Modulus of elasticity (E_L).* The modulus of elasticity is determined from the slope of the straight line portion of the graph, the steeper the line, the higher being the modulus. Modulus of elasticity is computed by

$$E_L = \frac{P_p L^3}{48 \delta_p I} = \frac{P_p L^3}{4 \delta_p b d^3} \tag{2:5}$$

The standard static bending test is made under such conditions that shear deformations are responsible for approximately 10 percent of the deflection. Values of E_L from tests made under such conditions and calculated by the formula shown do not, therefore, represent the true modulus of elasticity of the material, but an "apparent" modulus of elasticity.

The use of these values of apparent modulus of elasticity in the usual formulas will give the deflection of simple beams of ordinary length with but little error. For I- and box beams, where more exact computations are desired, and formulas are used that take into account the effect of shear deformations, a "true" value of the modulus of elasticity is necessary and may be had by adding 10 percent to the values in tables 2-6 and 2-7.

2.1201. *Fiber stress at proportional limit (F_{bp}).* The plotted points from which the early portions of the curves of figure 2-4 (a) were drawn lie approximately on a straight line, showing that the

deflection is proportional to the load. As the test progresses however, this proportionality between load and deflection ceases to exist. The

point at which this occurs is known as the proportional limit. The corresponding stress in the extreme fibers of the beam is known as "fiber stress at proportional limit." Fiber stress at proportional limit is computed by

$$F_{bp} = \frac{P_p L c}{4 I} = \frac{1.5 P_p L}{b d^2} \tag{2:6}$$

2.1202. *Modulus of rupture (F_{bu})*. Modulus of rupture is computed by the same formula as was used in computing fiber stress at proportional limit, except that maximum load is used in place of load at proportional limit. Since the formula used is based upon an assumption of linear variation of stress across the cross section of the beam, modulus of rupture is not truly a stress existing at time of rupture, but is useful in finding the load-carrying capacity of a beam.

2.1203. *Work to maximum load*. The energy absorbed by the specimen up to the maximum load is represented by the area under the load-deflection curve from the origin to a vertical line through the abscissa representing the maximum deflection at which the maximum load is sustained. It is expressed, in tables 2-6 and 2-7, in inch-pounds per cubic inch of specimen. Work to maximum load is computed by

$$\text{Work to } P_{\max} = \frac{\text{area under curve to } P_{\max}}{b \times d \times L} \tag{2:7}$$

2.121. *Compression parallel to grain*. In the compression-parallel-to-grain test, a 2- by 2- by 8-inch block is compressed in the direction of its length at a constant rate (0.024 inch per minute). The load is applied through a spherical bearing block, preferably of the suspended self-aligning type, to insure uniform distribution stress. On some of the specimens, the load and the deformation in a 6-inch central gage length are read simultaneously until the proportional limit is passed. The test is discontinued when the maximum load is passed and the failure appears.

Figure 2-4 (b) shows a test set-up, and typical load-deflection curves for Sitka spruce and yellow birch. Data on a number of properties are obtained from this test. These are discussed as follows:

2.1210. *Modulus of elasticity (E_L)*. The modulus of elasticity is determined from the slope of the straight-line portion of the graph, the steeper

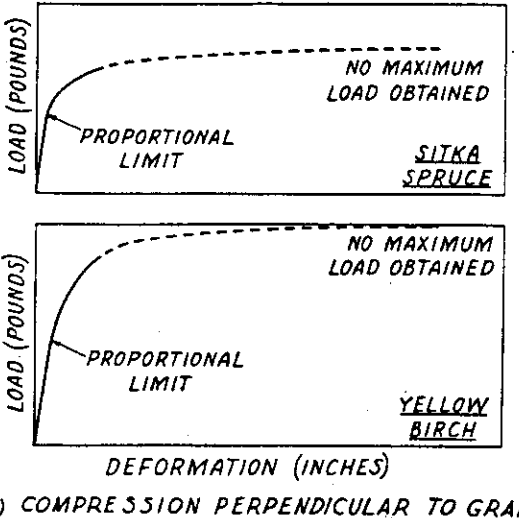
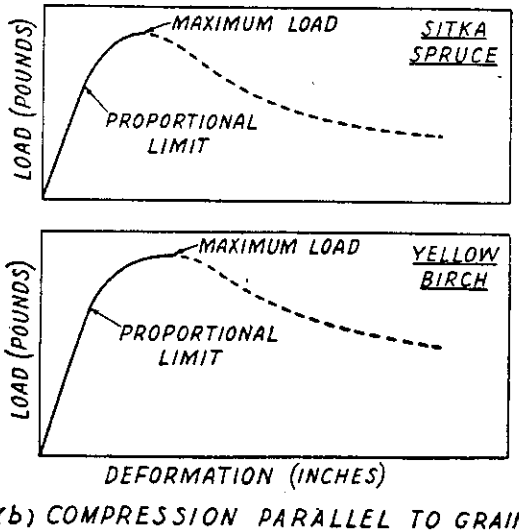
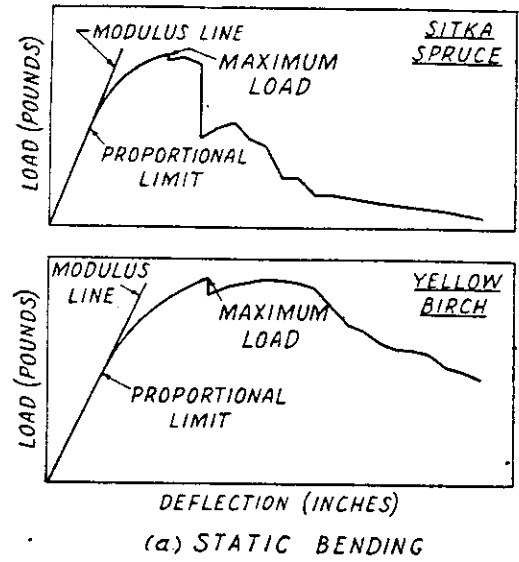


Figure 2-4. Standard test methods and typical load-deflection curves

the line the higher the modulus. The modulus of elasticity is computed by

$$E_{Lc} = \frac{P_p}{Ae_L} \quad (2:8)$$

The value of the modulus of elasticity so determined corresponds to the "true" value of modulus of elasticity discussed under static bending. Values of the modulus of elasticity from compression-parallel-to-grain tests are not published but may be approximated by adding 10 percent to the apparent values shown under static bending in table 2-6.

A multiplying factor of 1.1 has been inserted in various formulas throughout this bulletin to convert E_L values, as shown in tables 2-6 and 2-7 to E_{Lc} values required in formulas involving direct stress.

2.1211. *Fiber stress at proportional limit (F_{cp})*. The plotted points from which early portions of the curves of figure 2-4 (b) were drawn lie approximately on a straight line, showing that the deformation within the gage length is proportional to the load. The point at which this proportionality ceases to exist is known as the proportional limit and the stress corresponding to the load at proportional limit is the fiber stress at proportional limit. It is calculated by

$$F_{cp} = \frac{P_p}{A} \quad (2:9)$$

2.1212. *Maximum crushing strength (F_{cu})*. The maximum crushing strength is computed by the same formula as used in computing fiber stress at proportional limit except that maximum load is used in place of load at proportional limit.

2.122. *Compression perpendicular to grain*. The specimen for the compression-perpendicular-to-grain test is 2 by 2 inches in cross section and 6 inches long. Pressure is applied through a steel plate 2 inches wide placed across the center of the specimen and at right angles to its length. Hence, the plate covers one-third of the surface. The standard placement of the specimen is with the growth rings vertical. The standard rate of descent of the movable head is 0.024 inch per minute. Simultaneous readings of load and compression are taken until the test is discontinued at 0.1-inch compression.

Figure 2-4 (c) shows a test set-up, and typical load-deflection curves for Sitka spruce and yellow birch.

The principal property determined is the stress at proportional limit (F_{cp}) which is calculated by

$$F_{cp} = \frac{\text{Load at proportional limit}}{\text{Width of plate} \times \text{width of specimen}} \quad (2:10)$$

Tests indicate that the stress at proportional limit when the growth rings are placed horizontal does not differ greatly from that when the growth rings are vertical. For design purposes, therefore, the values of strength in compression perpendicular to grain as given in tables 2-6 and 2-7 may be used regardless of ring placement.

2.123. *Shear parallel to grain (F_{su})*. The shear-parallel-to-grain test is made by applying force to a 2- by 2-inch lip projecting $\frac{3}{4}$ inch from a block 2 $\frac{1}{2}$ inches long. The block is placed in a special tool having a plate that is seated on the lip and moved downward at a rate of 0.015 inch per minute. The specimen is supported at the base so that a $\frac{1}{8}$ -inch offset exists between the outer edge of the support and the inner edge of the loading plate.

The shear tool has an adjustable seat in the plate to insure uniform lateral distribution of the load. Specimens are so cut that a radial surface of failure is obtained in some and a tangential surface of failure in others.

The property obtained from the test is the maximum shearing strength parallel to grain. It is computed by

$$F_{su} = \frac{P_{max}}{A} \quad (2:11)$$

The value of F_{su} as found when the surface of failure is in a tangential plane does not differ greatly from that found when the surface of failure is in a radial plane, and the two values have been combined to give the values shown in column 14 of tables 2-6 and 2-7.

2.124. *Hardness*. Hardness is measured by the load required to embed a 0.444-inch ball to one-half its diameter in the wood. (The diameter of the ball is such that its projected area is one square centimeter.) The rate of penetration of the ball is 0.25 inch per minute. Two penetrations are made on each end, two on a radial, and two on a tangential surface of the specimen. A special tool makes it easy to determine when the proper penetration of the ball has been reached. The accompanying load is recorded as the hardness value.

Values of radial and tangential hardness as

Table 2-9. Elastic constants of various species

Species	Specific gravity ¹	Moisture content ¹	Young's modulus ratios		Modulus of rigidity ratios			Poisson's ratios					Source of data	
			E_T/E_L	E_R/E_L	G_{LR}/E_L	G_{LT}/E_L	G_{RT}/E_L	μ_{LR}	μ_{LT}	μ_{RT}	μ_{RL}	μ_{TL}		
Balsa ²	0.131	Percent 9.4	0.015	0.046	0.054	0.037	0.005	0.229	0.488	0.665	0.231	0.018	0.009	Reference 2-3. Reference 2-26. Do.
Douglas-fir	.506	7.5	.061	.090				.295	.441	.447	.368	.030	.023	
Do.	.506	12.9	.050	.068				.292	.449	.390	.374	.036	.029	Do.
Do.	.506	19.9	.032	.056				.274	.496	.560	.396	.018	.019	Do.
Do.	.45	11.2			.064	.078	.007							Reference 2-4.
Khaya	.45	11.3	.050	.111	.088	.059	.021	.297	.641	.604	.264	.033	.032	Reference 2-2.
Mahogany	.50	11.7	.064	.107	.086	.066	.028	.314	.533	.600	.326	.033	.034	Reference 2-2.
Quipo ³	.137	11.2	.055	.182	.082	.055	.032	.216	.666	.455	.128	.047	.032	Reference 2-3.
Sitka spruce	.378	7.1	.050	.089				.375	.436	.468	.248	.034	.022	Reference 2-10.
Do.	.378	12.8	.043	.078				.372	.467	.435	.245	.040	.025	Do.
Do.	.378	16.3	.036	.064				.374	.504	.527	.278	.030	.020	Do.
Do.	.378	21.6	.029	.053				.371	.539	.512	.278	.022	.017	Do.
Do.	.39	6.8			.070	.068	.004							Reference 2-5.
Do.	.39	11.1			.064	.061	.003							Do.
Sweetgum	.536	11.2	.050	.115				.325	.403	.682	.309	.044	.023	Reference 2-25.
Do.	.530	10.2			.089	.061	.021							
Yellow birch	.64	13.3	.050	.078				.426	.451	.697	.426	.043	.024	Reference 11.
Do.	.65	12.6			.074	.068	.017							Do.
Yellow-poplar	.376	10.7	.043	.092	.075	.069	.011	.318	.392	.703	.329	.030	.019	Reference 2-12.
Ash	.801	13.6	.064	.109	.057	.041	.0165	.533	.653	.656	.386	.0582	.0421	Reference 2-21.
Walnut	.593	11.0	.056	.106	.085	.062	.0209	.495	.632	.718	.367	.0520	.0360	Do.

¹ Specific gravity based on volume at test and weight when oven dry. The values shown to 2 decimal places, and corresponding moisture values, are tentative, and subject to revision.

² The balsa in these tests varied in specific gravity from 0.06 to 0.22, in which range E_L is given approximately by the equation $E_L = 5,500,000 \times \text{specific gravity} - 200,000$.

³ The quipo in these tests varied in specific gravity from 0.08 to 0.20, in which range E_L is given approximately by the equation $E_L = 3,260,000 \times \text{specific gravity} - 170,000$.

determined by the standard test have been averaged to give the values of side hardness in tables 2-6 and 2-7.

2.125. *Tension perpendicular to grain* (F_{tuT}). The tension-perpendicular-to-grain test is made to determine the resistance of wood across the grain to slowly applied tensile loads. The test specimen is 2 by 2 inches in cross section, and 2½ inches in overall length, with a length at midheight of 1 inch. The load is applied with special grips, the rate of movement of the movable head of the testing machine being 0.25 inch per minute. Some specimens are cut to give a radial and others to give a tangential surface of failure.

The only property obtained from this test is the maximum tensile strength perpendicular to grain. It is calculated from the formula

$$F_{tuT} = \frac{P_{maz.}}{A} \quad (2:12)$$

Tests indicate that the plane of failure being tangential or radial makes little difference in the strength in tension perpendicular to grain. Results from both types of specimens have, therefore, been combined to give the values shown in tables 2-6 and 2-7.

2.13. ELASTIC PROPERTIES NOT INCLUDED IN TABLES 2-6 AND 2-7. Certain elastic properties useful in design are not included in tables 2-6 and 2-7. The data in tables 2-6 and 2-7 are, in general, based on large numbers of tests, while the data on the additional elastic properties are based on relatively few tests. Available data on these properties are included in table 2-9.

2.130. *Moduli of elasticity perpendicular to grain* (E_T , E_R). The modulus of elasticity of wood perpendicular to the grain is designated as E_T when the direction is tangential to the annual growth rings, and E_R when the direction is radial to the annual growth rings. Tests have been made to evaluate these elastic properties for only a few species (table 2-9). The ratios $\frac{E_T}{E_L}$ and $\frac{E_R}{E_L}$ vary greatly among species and are considerably affected by differences in specific gravity and moisture content. For species not listed in the table, a rough approximation of the values for E_T and E_R may be made by assuming values of $\frac{E_T}{E_L}$ and $\frac{E_R}{E_L}$ as 0.05 and 0.10, respectively. Values of E_L are given in tables 2-6 and 2-7.

2.131. *Moduli of rigidity* (G_{LT} , G_{LR} , G_{RT}). The modulus of elasticity in shear, or the modulus of

rigidity as it is called, must be associated with shear deformation in one of the three mutually perpendicular planes defined by the L , T , and R directions, and with shear stresses in the other two. The symbol for modulus of rigidity has subscripts denoting the plane of deformation. Thus the modulus of rigidity G_{LT} refers to shear deformations in the LT plane resulting from shear stresses in the LR and RT planes. Values of these moduli for a few species are given in table 2-9. The ratios of G_{LT} , G_R , and G_{RT} to E_L vary among species and appear to be considerably affected by differences in specific gravity and moisture content. For species not listed in the table, it is recommended the approximate ratios $\frac{G_{LT}}{E_L}=0.06$, $\frac{G_{LR}}{E_L}=0.075$, and $\frac{G_{RT}}{E_L}=0.018$ be used in evaluating the various moduli of rigidity. The two letters of the subscript may be interchanged without changing the meaning of G .

2.132. *Poisson's ratios* (μ). The Poisson's ratio relating to the contraction in the T direction under a tensile stress acting in the L direction, and thus normal to the RT plane, is designated as μ_{LT} ; μ_{LR} , μ_{RT} , μ_{RL} , μ_{TR} , and μ_{TL} have similar significance, the first letter of the subscript in each relating to the direction of stress and the second to the direction of the lateral deformation.

Thus, the two letters of the subscript may not be interchanged without changing the meaning. The Poisson's ratios appear to be independent of specific gravity but are variously affected by differences in moisture content. Information on Poisson's ratios for wood is meager and values for only a few species are given in table 2-9.

2.14. STRESS-STRAIN RELATIONS. For most practical purposes wood can be considered to be an orthotropic material having orthotropic axes L , T , and R (see sec. 2.00). If the directions of the applied stresses are parallel to a plane containing two of these axes, the methods described in sections 2.56 to 2.5602, inclusive, can be applied. The general equations for stresses applied in any direction can be obtained from reference 2-53.

2.15. STRENGTH UNDER MULTIAXIAL STRESS. If the directions of the applied stresses are parallel to a plane containing two of the orthotropic axes, the methods described in section 2.610 to 2.613, inclusive, for plywood can be applied. The ultimate shear stress associated with relative shear displacements of the L and R axes and the L and T axes are each equal to the ultimate shear stress

parallel to the grain (F_{su}). The ultimate shear stress associated with relative shear displacements of the R and T axes is for hardwoods approximately one-half and for coniferous woods one-third of F_{su} (ref. 2-48). The ultimate compressive stress

associated with the T and R axes are each equal to the compressive strength perpendicular to the grain (F_{suT}) and the ultimate tensile stress associated with these axes are each equal to the tensile strength perpendicular to the grain (F_{tuT}).

The general equation defining the condition of failure for stresses applied in any direction is similar to equation (2:51) except that its left hand member contains six terms instead of three. If the ultimate stresses are given the values indicated in the preceding paragraph this equation can be written in the following form:

$$\left(\frac{f_L}{F_L}\right)^2 + \left(\frac{f_T}{F_T}\right)^2 + \left(\frac{f_R}{F_R}\right)^2 + \frac{(f_{LT})^2 + (f_{LR})^2 + (K f_{RT})^2}{(F_{su})^2} = 1 \quad (2:13)$$

in which f_L , f_T , and f_R are the three internal direct stresses in the directions of the axes L , T , and R , respectively, and f_{LT} , f_{LR} , and f_{RT} are the three internal shear stresses associated with shear displacements of the L and T axes, the L and R axes, and the R and T axes, respectively. Also F_L , F_T , and F_R are the three ultimate stresses associated with the directions of the L , T , and R axes, respectively, and may be tensile or compressive; thus F_L is tensile if f_L is tensile and compressive if f_L is compressive and similarly for F_T and F_R . Thus F_L is equal to F_{cu} or F_{tu} ; F_T is equal to F_{cuT} or F_{tuT} ; and F_R is equal to F_{cuT} or F_{tuT} . The value of K is 2 for hardwoods and 3 for coniferous woods.

Equation (2:13) can be handled in the manner described in section 2.613 and in reference 2-67 using the transformation equations for three dimensions given in reference 2-53.

The methods described in this section have not been verified by test but their verification for plywood indicates that they probably will yield reasonable values. Also equation 2:44 has been compared with results of tests on solid wood in which the specimens were constrained by the testing equipment and good agreement was found, however, shear stress associated with relative shear displacements of the R and T axes was not involved in these tests.

2.16. STRESS CONCENTRATIONS. Wood has

plastic as well as elastic properties (see sec. 2.06 on creep) and, therefore, stress concentrations in tension, compression, or shear are greatly relieved with the passage of time. In compression and shear, creep is very rapid for stresses near the ultimate value and, therefore, values of stress concentration calculated by means of the mathematical theory of elasticity are rarely attained. Creep at high stress in tension, however, is not nearly so rapid and calculated values of stress concentration may be approximately correct if the load is suddenly applied. This fact should be given careful consideration in the design of wood and plywood tension members, and stress concentrations should be avoided.

2.161. *Stress concentrations around a hole in a tension or compression member.* Figure 2-5 shows a panel of wood pierced by an elliptical hole which is small compared to the size of the panel. Axes y and z are orthotropic axes of the wood as well as axes of the ellipse. When a tensile stress (f_a) is applied as shown in the figure, tensile stress concentrations occur at the ends of axis a . The value of the stress at these points is given by equation (2:14):

$$f = f_a \left(\frac{a}{b} \sqrt{\frac{E_y}{G_{yz}} - 2\mu_{yz}} + 2 \sqrt{\frac{E_y}{E_z}} + 1 \right) \quad (2:14)$$

in which f_a denotes the value of the applied stress, subscript y denotes the orthotropic axis which is parallel to the stress, and subscript z denotes the other orthotropic axis which lies in the plane of the panel. These subscripts may represent the L , T , or R directions depending upon the directions of the grain and annual rings in the panel. Equation (2:14) applies also to a compressive stress.

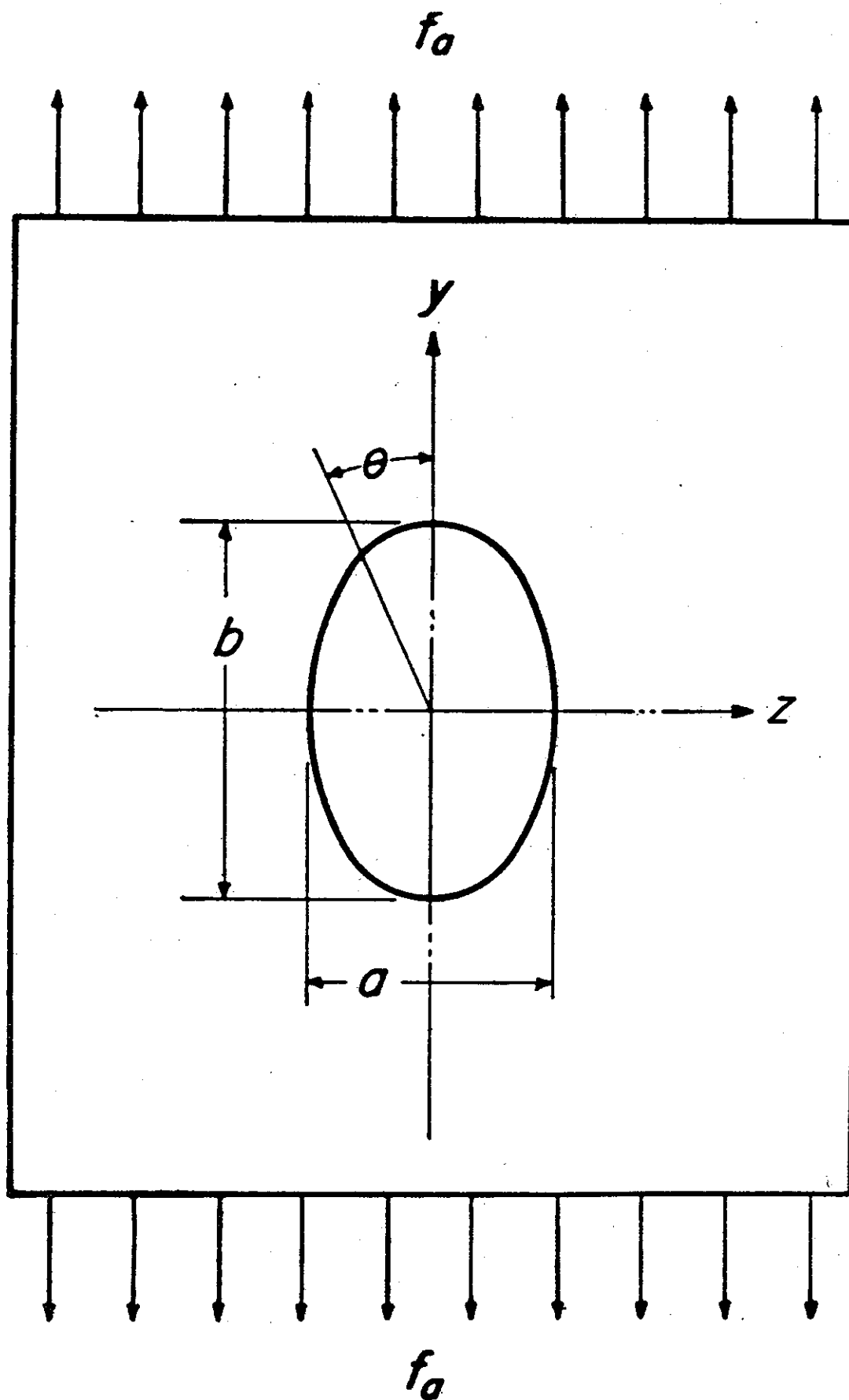


Figure 2-5. A wood or plywood panel pierced by an elliptical hole.

The shear stress on the periphery of the ellipse associated with relative shear displacements of axes y and z is given by equation (2:15):

$$f_s = f_a \frac{\frac{a}{b} \sqrt{\frac{E_y}{E_z}} \cos^2 \theta - \left[\sqrt{\frac{E_y}{G_{yz}} - 2\mu_{yz}} + 2 \sqrt{\frac{E_y}{E_z} + \frac{b}{a}} \right] \sin^2 \theta}{\left(\frac{a}{b}\right)^2 \frac{E_y}{E_z} \cos^4 \theta + \left[\frac{E_y}{G_{yz}} - 2\mu_{yz} \right] \sin^2 \theta + \cos^2 \theta \left(\frac{b}{a}\right)^2 \sin^4 \theta} \sin \theta \cos \theta \quad (2:15)$$

By use of equation (2:15) the shear stress can be plotted against θ and its maximum value found. For a plane sawed Sitka spruce panel pierced by a circular hole ($a=b$) a maximum value of $0.71 f_a$ was found at $\theta=78^\circ$ (see ref. 2-69).

Equations (2:14) and (2:15) can be used for plywood if the subscripts y and z are replaced by the subscripts b and a , respectively.

2.162. *Stress concentration due to a hole which is not small compared to the size of the member.* Stress concentrations in isotropic materials around holes that are not small compared to the size of the members pierced by them have been determined by photoelastic methods and are well known. It is impossible to use such methods in connection with wood, however, an estimate of the stress concentrations can be obtained by use of equation (2:14).

For an isotropic material equation (2:14) reduces to equation (2:16)

$$f = f_a \left(2 \frac{a}{b} + 1 \right) \quad (2:16)$$

An approximate corrective factor for use with the stress concentrations obtained for isotropic materials to obtain those for wood, or plywood, can be obtained by dividing values obtained by equation (2:14) by those obtained by equation (2:16). Of course such corrections do not apply to the shear stresses such as those obtained by equation (2:15).

2.2. Columns

2.20. PRIMARY FAILURE. The allowable stresses for solid wood columns are given by the following formulas:

Long columns

$$F_c = \frac{10 E_L}{\left(\frac{L'}{\rho}\right)^2} \text{ psi}$$

$$\left(\frac{L'}{\rho}\right)_{cr} = \sqrt{\frac{15 E_L}{F_{cu}}} \quad (2:17)$$

Short columns (ref. 2-97)

$$F_c = F_{cu} \left[1 - \frac{1}{3} \left(\frac{L'}{K\rho}\right)^4 \right] \text{ psi} \quad (2:18)$$

where

$$K = \left(\frac{L'}{\rho}\right)_{cr}$$

These formulas are reproduced graphically in figure 2-6 for solid wood struts of a number of species.

2.21. LOCAL BUCKLING AND TWISTING FAILURE. The formulas given in section 2.20 do not apply when columns with thin outstanding flanges or low torsional rigidity are subject to local buckling or twisting failure. For such cases, the allowable stresses are given by the following formulas:

Local buckling (torsionally rigid columns)

$$F_c = 0.07 E_L \left(\frac{t}{b}\right)^2 \text{ psi} \left(\text{when } \frac{b}{t} > 6\right) \quad (2:19)$$

Twisting failure (torsionally weak columns)

$$F_c = 0.044 E_L \left(\frac{t}{b}\right)^2 \text{ psi} \left(\text{when } \frac{b}{t} > 5\right) \quad (2:20)$$

When the width-thickness ratio (b/t) of the outstanding flange is less than the values noted, the column formulas of section 2.20 should be used. Failure due to local buckling or twisting can occur only when the critical stress for these types of failure is less than the stress required to cause primary failure. For unconventional shapes, tests should be conducted to determine suitable column curves (ref. 2-79).

2.22. LATERAL BUCKLING. When subjected to axial compressive loads, beams will act as columns tending to fail through lateral buckling. The usual column formulas (2:17 and 2:18) will apply except that when two beams are interconnected by ribs so that they will deflect together (laterally), the total end load carried by both beams will be the sum of the critical end loads for the individual beams.

The column lengths will usually be the length of a drag bay in a conventional wing. A restraint

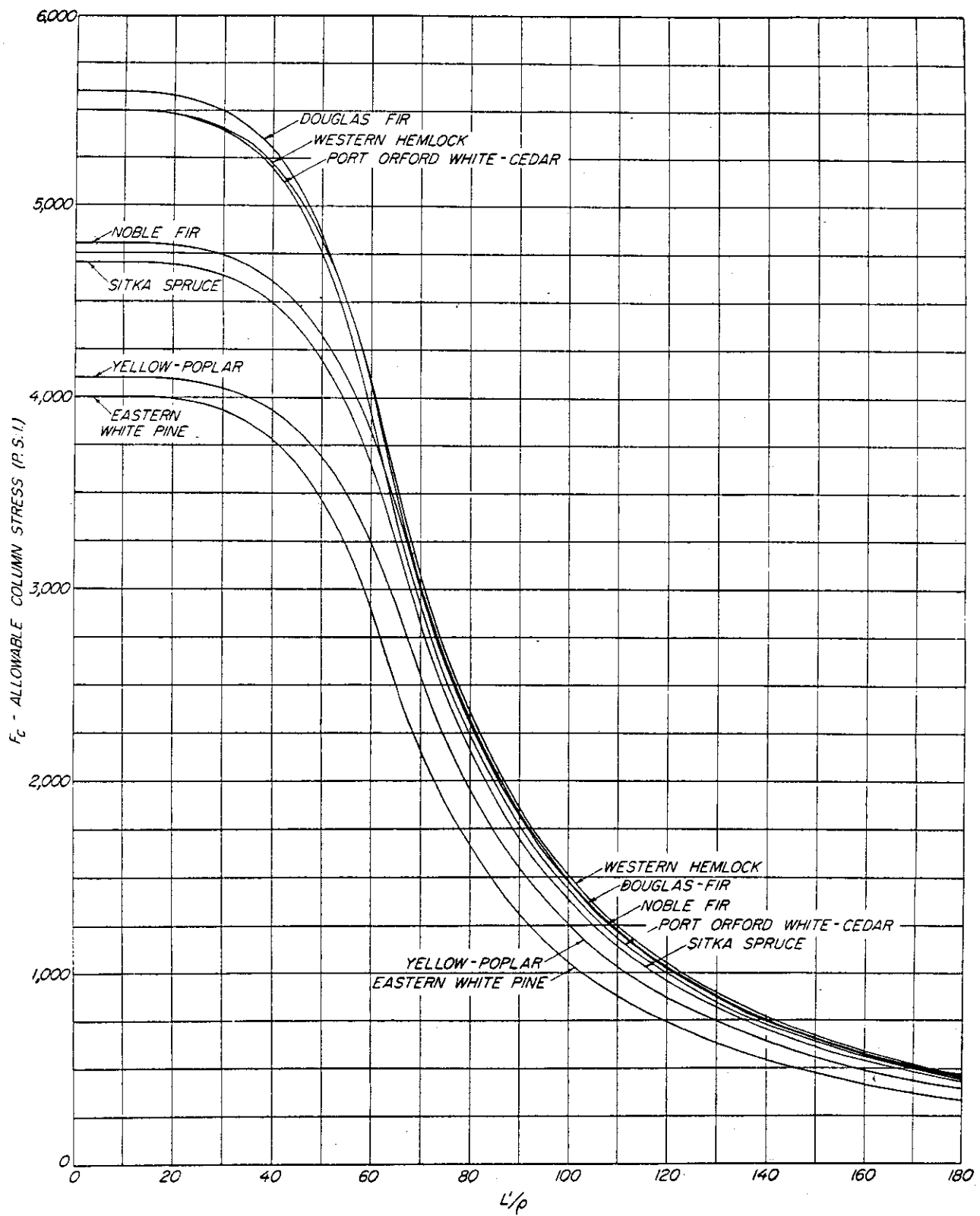


Figure 2-6. Allowable column stresses for solid wood struts.

coefficient of 1.0 will be applicable unless the construction is such that additional restraint is afforded by the leading edge or similar parts. Certain rules for such conditions will be found in the requirements of the certificating or procuring agencies.

2.3. Beams

2.30. FORM FACTORS. When other than solid rectangular cross sections are used for beams (I-beams or box beams), the static-bending strength properties given in table 2-6 must be multiplied by a "form factor" for design purposes. This form factor is the ratio of either the fiber stress at proportional limit or the modulus of rupture (in bending) of the particular section to the same property of a standard 2-inch square specimen of that material. The proportional limit form factor (FF_p) is given by the formula:

$$FF_p = 0.58 + 0.42 \left(K \frac{b-b'}{b} + \frac{b'}{b} \right) \quad (2:21)$$

and the modulus of rupture form factor (FF_u) by the formula:

$$FF_u = 0.50 + 0.50 \left(K \frac{b-b'}{b} + \frac{b'}{b} \right) \quad (2:22)$$

where

b' = total web thickness

b = total flange width (including any web(s))

K = constant obtained from figure 2-7

Formulas 2:21 and 2:22 cannot be used to determine the form factors of sections in which the top and bottom edges of the beam are not perpendicular to the vertical axis of the beam. In such cases, it is first necessary to convert the section to an equivalent section whose height equals the mean height of the original section, and whose width and flange areas equal those of the original section, as shown in figure 2-7. The fact that the two beams of each pair shown in figure 2-7 developed practically the same maximum load in test demonstrates the validity of this conversion (ref. 2-56 and 2-62).

Tests have indicated that the modulus of rupture which can be developed by a beam of rectangular cross section decreases with the height. Sufficient data are not available to permit exact evaluation of the reduction as the height increases, but where deep beams of rectangular cross section are to be used, thought should be given to the

reduction of the value for modulus of rupture given in tables 2-6 and 2-7.

2.31. TORSIONAL INSTABILITY. It is possible for deep thin beams to fail through torsional instability at loads less than those indicated by the usual beam formula. Reference 2-63 gives formulas for calculating the strength of such beams for various conditions of end restraint. However, in view of the difficulty of accurately evaluating the modulus of rigidity and end fixity, it is always advisable to conduct static tests of a typical specimen. This will apply to cases in which the ratio of the moment of inertia about the horizontal axis to the moment of inertia about the vertical axis exceeds approximately 25 (ref. 2-62 and 2-63).

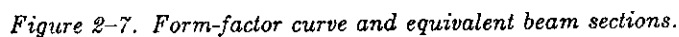
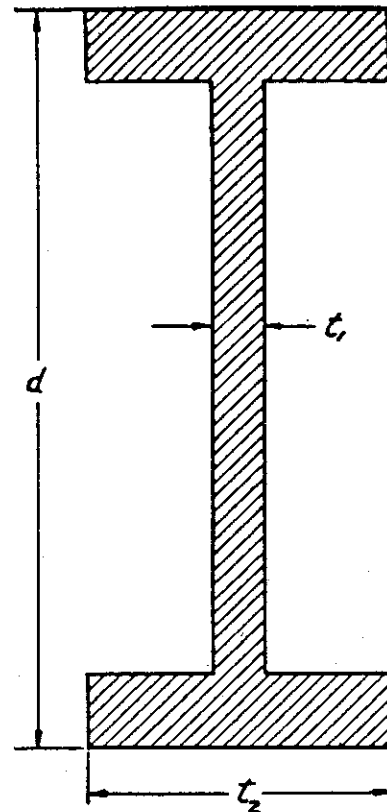
2.32. COMBINED LOADINGS.

2.320. *General.* Because of the variation of the strength properties of wood with the direction of loading with respect to the grain, no general rules for combined loadings can be presented, other than those for combined bending and compression given in section 2.321, and those for combined bending and tension given in section 2.322. When unusual loading combinations exist, static tests should be conducted to determine the desired information.

2.321. *Bending and compression.* When subjected to combined bending and compression, the allowable stress for spruce, Western hemlock, and noble fir beams at 15 percent moisture content can be determined from figure 2-8 and that for Douglas-fir beams from figure 2-9. The charts are based on a method of analysis developed by the Forest Products Laboratory (ref. 2-63 and 2-78).

The curves of figures 2-8 and 2-9 are based on the use of a fourth-power parabola for columns of intermediate length. On these figures the horizontal family of curves indicates the proportional limit under combined bending and compression; the vertical family, the effect of various slenderness ratios on bending. The allowable stress, F_{bc} , under combined load is found as follows:

- (1) For the cross section of a given beam, find the proportional limit in bending and the modulus of rupture from the ratios of compression-flange thickness to total depth and of web thickness to total width, locating such points as *A* and *B*.
- (2) Project points *A* and *B* to the central line, obtaining such points as *C* and *D*.



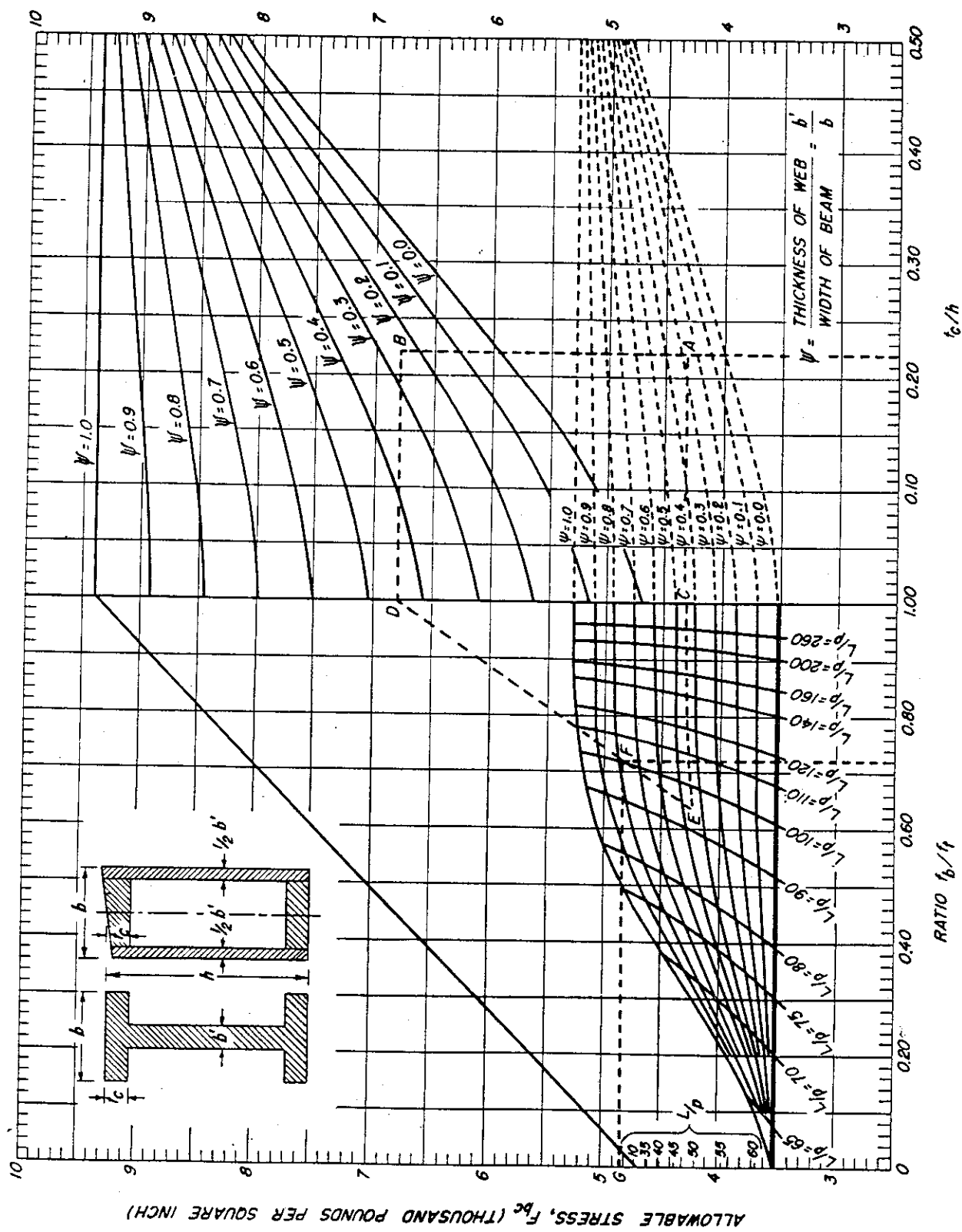


Figure 2-8. Allowable stresses for Sitka spruce spars subjected to combined bending and compression at 15 percent moisture content.

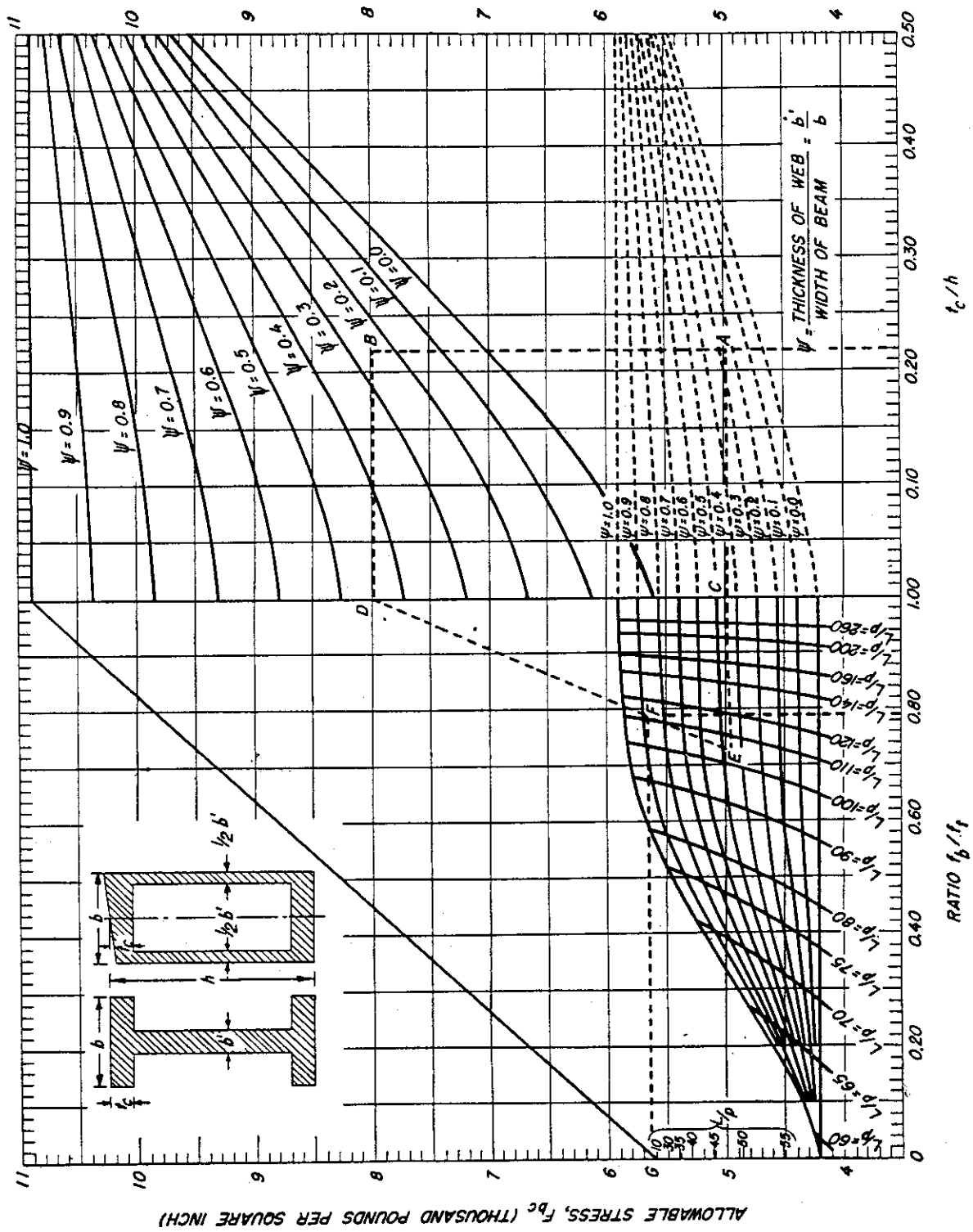


Figure 2-9. Allowable stresses for Douglas-fir spars subjected to combined bending and compression at 15 percent moisture content.

- (3) Locate a point, such as E , indicating the proportional limit of the given section under combined bending and compression. This point will be at the intersection of the curve of the "horizontal" family through C and the curve of the slenderness ratio corresponding to the distance between points of inflection.

(4) Draw line ED .

- (5) Locate point F on line ED , with an abscissa equal to the computed ratio of bending to total stress. The ordinate of F represents the desired value of the allowable stress.

The following rules should be observed in the use of figures 2-8 and 2-9:

- (1) The length to be used in computing the slenderness ratio, L/ρ should be determined as follows:
 - (a) If there are no points of inflection between supports, L should be taken as the distance between supports.
 - (b) If there are two points of inflection between supports, L should be taken as the distance between these points of inflection when calculating the allowable strength of any section included therein.
 - (c) When calculating the allowable strength of a section between a point of inflection and an intermediate support of a continuous beam, L should be taken as the distance between the points of inflection adjacent to the support on either side.
 - (d) When investigating a section adjacent to an end support, L should be taken as twice the distance between the support and the adjacent point of inflection, except that it need not exceed the distance between supports.
- (2) In computing the value of ρ for use in determining the slenderness ratio, L/ρ , filler blocks should be neglected and, in the case of tapered spars, the average value should be used.
- (3) In computing the modulus of rupture and the proportional limit in bending, the properties of the section under investigation should be used. Filler blocks may be included in the section for this purpose. When computing the form factor of box

spars the total thicknesses of both webs should be used.

2.322. *Bending and tension.* When tensile axial loads exist, the maximum computed stress on the tension flange should not exceed the modulus of rupture of a solid beam in pure bending. Unless the tensile load is relatively large, the compression flange should also be checked, using the modulus of rupture corrected for form factor.

2.33. *SHEAR WEBS.* See section 2.73.

2.34. *BEAM SECTION EFFICIENCY.* In order to obtain the maximum bending efficiency of either I or box beams, the unequal flange dimensions can be determined by first designing a symmetrical beam of equal flanges. The amount of material to be transferred from the tension side to the compression side, keeping the total cross-sectional area, height, and width constant, is given by the following equation (ref. 2-62):

$$x = \frac{Abh^2 - \sqrt{A^2b^2h^4 - 4AI_s bhwD}}{2wDbh} \quad (2:23)$$

where

A = total area of the cross section

b = total width

h = total depth

w = width of flange

D = clear distance between flanges

I_s = moment of inertia of the symmetrical section

x = thickness to be taken from tension flange and added to compression flange

In using this equation, the following procedure is to be followed:

- (a) Determine the section modulus required.
- (b) Determine the sizes of flanges of equal size to give the required section modulus.
- (c) Using equation (2:23), compute the thickness of material to be transferred from the tension flange to the compression flange. The procedure thus far will result in a section modulus greater than required. To obtain a beam of the required section modulus, either (d) or (e) may be followed.
- (d) Calculate the ratio of depth of tension flange to compression flange and design a section having flanges with this ratio and the required section modulus, or
- (e) Carry out steps (a), (b), and (c) starting with a symmetrical section having a section modulus less than that required

until an unsymmetrical section having the required section modulus is obtained.

- (f) Beams designed according to the foregoing procedure should always be checked for adequacy of glue area between webs and tension flange. This consideration may govern the thickness of the tension flange.

2.4. Torsion

2.40. GENERAL. The torsional deformation of wood is related to the three moduli of rigidity, G_{LT} , G_{LR} , and G_{RT} . When a member is twisted about an axis parallel to the grain, G_{RT} is not involved; when twisted about an axis radial to the grain direction, G_{LT} is not involved; when twisted about an axis tangential to the grain direction, G_{LR} is not involved. No general relationship has

been found for the relative magnitudes of G_{LR} , G_{LT} , and G_{RT} (table 2-9).

2.41. TORSIONAL PROPERTIES. The "mean modulus of rigidity" (G) taken as 1/16 of E_L , may be safely used in the standard formulas for computing the torsional rigidities and internal shear stresses of solid wood members twisted about an axis parallel to the grain direction. Torsion formulas for a number of simple sections are given in table 2-10. For solid-wood members the allowable ultimate torsional shear stress (F_{st}) may be taken as the allowable shear stress parallel to the grain (column 14 in tables 2-6 and 2-7) multiplied by 1.18: that is, $F_{st}=1.18 F_{su}$. The allowable torsional shear stress at the proportional limit may be taken as two-thirds of F_{st} . The torsional strength and rigidity of box beams having plywood webs are given in section 2.75.

Table 2-10. Formulas for torsion on symmetrical sections

Section	Angle of twist in radians	Maximum shear stress
Circle.....	$\theta=\frac{32TL}{G\pi D^4}$	$f_s=\frac{16T}{\pi D^3}$
Circular tube.....	$\theta=\frac{TL}{GI_p}$	$f_s=\frac{TD}{2I_p}$
Ellipse ¹	$\theta=\frac{TL(a+b)}{G\pi a^3b^3}$	$f_s=\frac{2T}{\pi ab^2}$ at ends of short diameter
Square ²	$\theta=\frac{TL}{2.25Ga^4}$ (approx.)	$f_s=\frac{3T}{5a^3}$ (approx.)
Rectangle ³	$\theta=\frac{40I_pTL}{A^4G}$ (approx.)	$f_s=\frac{T(15a+9b)}{40a^2b^2}$ at midpoint of long side

¹ 2a=major axis; 2b=minor axis.
² 2a=side of square.
³ 2a=long side, 2b=short side.

2.5. Basic Strength and Elastic Properties of Plywood

2.50. GENERAL. Plywood is usually made with an odd number of sheets or plies of veneer with the grain direction of adjacent plies at right angles. Depending upon the method by which the veneer is cut, it is known as rotary-cut, sliced, or sawed veneer. Generally, the construction is symmetrical; that is, plies of the same species, thickness, and grain direction are placed in pairs at equal distances from the central ply. Lack of symmetry results in twisting and warping of the finished panel. The disparity between the properties of wood in directions parallel to and across the grain is reduced by reason of the arrangement of the

material in plywood. By placing some of the material with its strong direction (parallel to grain) at right angles to the remainder, the strengths in the two directions become more or less equalized. Since shrinkage of wood in the longitudinal direction is practically negligible, the transverse shrinkage of each ply is restrained by the adjacent plies. Thus, the shrinking and swelling of plywood for a given change in moisture content is less than for solid wood.

The tendency of plywood to split is considerably less than for solid wood as a result of the cross-banded construction. While many woods are cut into veneer, those species which have been approved for use in aircraft plywood are listed in table 2-11.

2.51. ANALYSIS OF PLYWOOD STRENGTH PROPERTIES. The analysis of the strength and elastic properties of plywood is complicated by the fact that the elastic moduli of adjacent layers are different. This is illustrated in figure 2-10 for bending of a three-ply panel. Assuming that strain is proportional to distance from the neutral axis, stresses on contiguous sides of a glue joint will be different by reason of the difference in the modulus of elasticity in adjacent layers. This results in a distribution of stress across the cross section as shown in figure 2-10 (c). Similar irregular stress distribution will be obtained for plywood subjected to other types of loading.

From this it may be seen that the strength and elastic properties of plywood are dependent not only upon the strength of the material and the dimensions of the member, as for a solid piece, but also upon the number of plies, their relative thickness, and the species used in the individual plies. In addition, plywood may be used with the direction of the face plies at angles other than 0° or 90° to the direction of principal stress and, in special cases, the grain direction of adjacent plies may be oriented at angles other than 90°.

In general, plywood for aircraft use has the grain direction (the longitudinal direction) of adjacent plies at right angles. The strength and elastic properties of the plywood are dependent upon the properties of solid wood along and across the grain as illustrated in figure 2-11.

Considerable information (tables 2-6 and 2-7) on the properties of wood parallel to the grain is

available, but the data on properties across the grain are less complete. Sufficient data are available, however, so that the elastic properties of wood in the two directions can be related with reasonable accuracy to the plywood properties. On this basis formulas are given which will enable the designer, knowing the number, relative thickness and species of plies, to compute the properties of plywood from the data given in tables 2-6 and 2-7.

The formulas given are only for plywood having the grain direction of adjacent plies at right angles and are applicable only to certain directions of stress. The limitations on the angle between the face grain and the direction of principal stress have been noted in each section. The formulas are intended for use only in these cases, and the interpolation must not be used to obtain values for intermediate angles unless specific information on these angles is given. Computed values of certain of the strength and elastic properties for many of the commonly used species and constructions of plywood are given in section 2.54, based on strength of wood at 15 percent moisture content (table 2-6).

2.52. BASIC FORMULAS. For purposes of discussion, plywood structural shapes may be conveniently separated into two groups: (a) elements acting as prisms, columns, and beams, and (b) panels. The fundamental difference between these two groups is that, in group (a) the plywood is supported or restrained only on two opposite edges, while in group (b) the plywood is supported

Table 2-11. Veneer species for aircraft plywood

Group I (high density) ^{1 2}	Group II (medium density) ²	Group III (low density) ³
American beech.....	Birch (Alaska and paper).....	Basswood.
Birch (sweet and yellow).....	Khaya species (so-called "African mahogany").	Yellow-poplar.
Maple (hard).....	Southern magnolia.....	Port Orford White-cedar.
Pecan.....	Mahogany (from tropical America).	Spruce (red, Sitka, and white) (quarter-sliced).
	Maple (soft).....	Ponderosa pine (quarter-sliced).
	Sweetgum.....	Sugar pine.
	Water tupelo.....	Noble fir (quarter-sliced).
	Black walnut.....	Western hemlock (quarter-sliced).
	Douglas-fir (quarter-sliced).....	Redwood (quarter-sliced).
	American elm (quarter-sliced).....	
	Sycamore.....	

¹ Where hardness, resistance to abrasion, and high strength of fastening are desired, Group I woods should be used for face stock.

² Where finish is desired, or where the plywood is to be steamed and bent into a form in which it is to remain, species of Group I and II should be used.

³ Group III species are used principally for core stock and cross-banding. However, where high bending strength or freedom from buckling at minimum weight is desired, plywood made entirely from species of Group III is recommended.

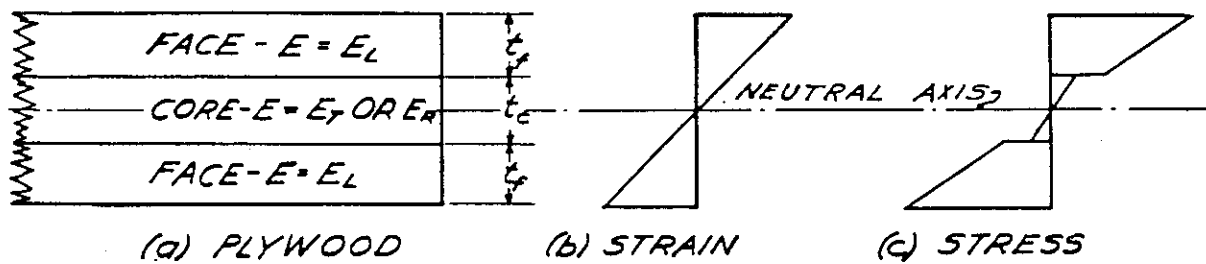


Figure 2-10. Three-ply plywood beam in bending.

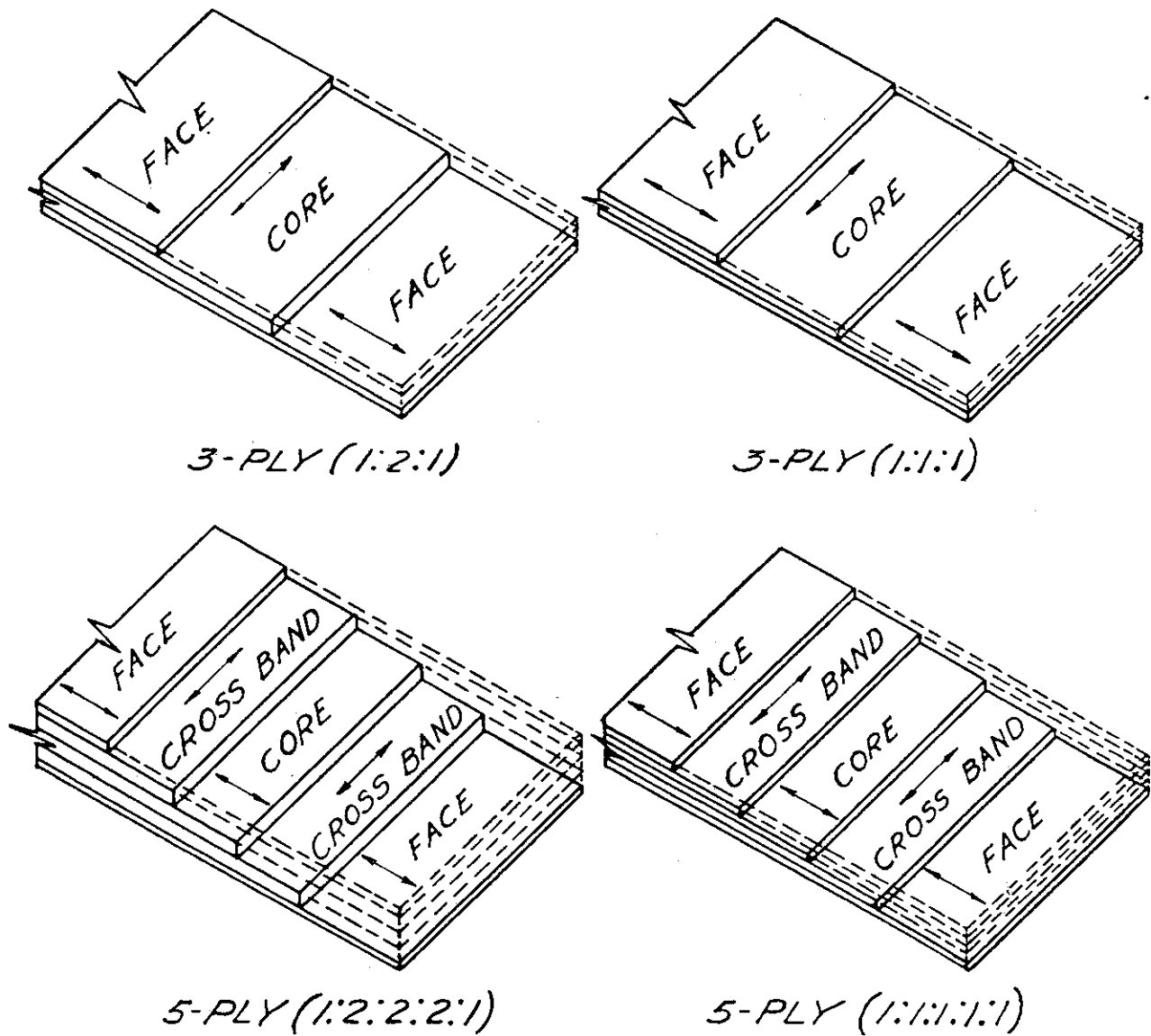


Figure 2-11. Typical plywood constructions. Arrows indicate grain direction of each ply.

or restrained on more than two edges. It is essential that this fundamental difference between the two groups be kept in mind during the application of the formulas¹ given here and in later sections (ref. 2-25, 2-43, and 2-54).

- (1) The effective moduli of elasticity of plywood in tension or compression are:

E_a —measured parallel to side a for panels (sec. 2.712, 2.713)

E_b —measured perpendicular to side a for panels (sec. 2.712, 2.713)

E_w —measured parallel to (with) the face grain

E_x —measured perpendicular to (across) the face grain, and are determined as

$$= \frac{1}{t} \sum_{i=1}^{i=n} E_i t_i \quad (2:24)$$

where

t —total thickness of plywood

t_i —thickness of i^{th} ply

E_i —modulus of elasticity of i^{th} ply measured in the same direction as the pertinent desired E (as E_a , E_b , E_w , or E_x). The value of E_i is equal to E_{Lc} (1.1 E_L from tables 2-6 and 2-7), or E_T , or E_R , (table 2-9) as applicable.

- (2) The effective moduli of elasticity of plywood in bending are:

E_1 —measured parallel to side a for panels

E_2 —measured perpendicular to side a for panels

E_{fw} —measured parallel to (with) the face grain

E_{fx} —measured perpendicular to (across) the face grain, and are determined as

$$f_s = \frac{1}{I} \sum_{i=1}^{i=n} E_i I_i \quad (2:25)$$

where

E_i —as defined under (1)

I —moment of inertia of the total cross section about the centerline, measured in the same direction as the pertinent desired E (namely, E_1 , E_2 , E_{fw} , or E_{fx}).

¹ When computing the various moduli of elasticity for plywood of balanced construction and all plies of the same species, the following relationship will be found helpful:

$$E_L + E_T = E_a + E_b = E_1 + E_2 = E_w + E_x = E_{fw} + E_{fx}$$

If the veneers are quarter-sliced rather than rotary-cut, the term E_T should be replaced by E_R .

I_i —moment of inertia of the i^{th} ply about the neutral axis of the same total cross section. For symmetrically constructed plywood, the neutral axis to be used in determining I_i will be the centerline of the cross section. For unsymmetrical plywood constructions, the neutral axis is usually not the centerline of the geometrical section. In this case the distance from this neutral axis to the extreme compression fiber is given by the equation:

$$c = \frac{\sum_{i=1}^{i=n} t_i E_i c_i}{\sum_{i=1}^{i=n} t_i E_i} \quad (2:26)$$

where

c_i —distance from the extreme compression fiber to the center of the i^{th} ply.

- (3) In calculating the bending strength (not stiffness) of plywood strips in bending having the face grain direction perpendicular to the span, a modulus E'_{fx} , similar to E_{fx} is to be used. For plywood made of five or more plies, the use of E_{fx} for E'_{fx} in strength calculations will result in but relatively small error. The value of E'_{fx} may be calculated in the same manner as that used in calculating E_{fx} except that the effect of the outer ply on the tension side is neglected. The location of the neutral axes used in calculating E_{fx} and E'_{fx} will be different. The value of E'_{fx} may also be calculated from the following formula:

$$E'_{fx} = E_{fx} + \frac{12 E_x}{t^2} \left(c' - \frac{t}{2} + t_f \right)^2 - \frac{12 t_f E_T}{t^3} \left(c' + \frac{t_f}{2} \right)^2 \quad (2:27)$$

where

$$c' = \frac{1}{2} \left[\frac{\frac{t E_x}{t_f E_T} (t - 2 t_f) + t_f}{\frac{t E_x}{t_f E_T} - 1} \right]$$

= distance from neutral axis to extreme fiber of the outermost longitudinal ply. E_T pertains to the species of the face ply.

- (4) The modulus of rigidity (modulus of elasticity in shear) of solid wood involves the shear moduli G_{LT} , G_{LR} , and G_{RT} .

As mentioned in section 2.131, little information is available on this elastic property, and a "mean" modulus of rigidity is ordinarily used for wood. Similarly for plywood, a value of modulus of rigidity based on the "mean" modulus of rigidity for solid wood may be used.

For plywood (all plies the same species) having the facegrain parallel or perpendicular to the direction of principal shearing stress, the modulus of rigidity may be taken the same as for solid wood.

The theoretical treatment of the elastic properties of plywood involves the moduli of rigidity G_{wx} and G_{fwx} . They are determined as:

$$G_{wx} = \frac{1}{t} \sum_{i=1}^{i=n} G_i t_i \quad (2:28)$$

$$G_{fwx} = \frac{1}{I} \sum_{i=1}^{i=n} G_i I_i \quad (2:29)$$

where the summations are taken over all plies in a section perpendicular to either the a or b directions using the modulus of rigidity in each ply in the wx plane.

When the plywood is made of a single species of wood,

$$G_{fwx} = G_{wx} = G_{LT} \text{ for rotary-cut veneer.}$$

$$G_{fwx} = G_{wx} = G_{LR} \text{ for quarter-sliced veneer.}$$

- (5) Poisson's ration (μ). Although there is very little information available on the values of Poisson's ratios for plywood, a brief summary of their significance is given.

The effective Poisson's ratio of plywood in tension or compression (no flexure) is the ratio of the contraction along the x direction to extension along the w direction due to tensile stress acting in the w direction and thus normal to the xt plane, or

$$\mu_{wx} = \frac{1}{t E_x} \sum_{i=1}^{i=n} t_i (E_x)_i (\mu_{wx})_i \quad (2:30)$$

where

$(E_x)_i$ = modulus of elasticity of the i^{th} ply in the x direction.

$(\mu_{wx})_i$ = Poisson's ratio of contraction along the x direction to extension in the w

direction due to a tensile stress acting the w direction and thus normal to the xt plane of the i^{th} ply.

Similarly

$$\mu_{zw} = \frac{1}{t E_w} \sum_{i=1}^{i=n} t_i (E_w)_i (\mu_{zw})_i \quad (2:31)$$

If all plies are of the same species of rotary-cut veneer

$$\mu_{wx} = E_L \mu_{TL} / E_x$$

$$\mu_{zw} = E_L \mu_{TL} / E_w$$

If all plies are of the same species of quarter-sliced veneer

$$\mu_{wx} = E_L \mu_{RL} / E_x$$

$$\mu_{zw} = E_L \mu_{RL} / E_w$$

These formulas give close approximations of the apparent Poisson's ratios in these two directions when the stress is simple tension or compression. For more accurate formulas than (2:30) and (2:31) see reference 2-65. For Poisson's ratio at an angle to the grain see section 2.56.

The Poisson's ratios associated with flexure are

$$\mu_{fwx} = \frac{1}{I E_{fx}} \sum_{i=1}^{i=n} I_i (E_x)_i (\mu_{fwx})_i \quad (2:32)$$

$$\mu_{fzw} = \frac{1}{I E_{fw}} \sum_{i=1}^{i=n} I_i (E_w)_i (\mu_{fzw})_i \quad (2:33)$$

These equations yield values identical to those of equations (2:30) and 2:31) if all plies of the plywood are of the same species of wood.

2.53. APPROXIMATE METHODS FOR CALCULATING PLYWOOD STRENGTHS. Table 2-12 gives some approximate methods of calculating the various strength properties of plywood. These simplified methods will be found very useful in obtaining estimates on the strength of plywood, but cannot be relied upon to give results that are comparable to those obtained with the more accurate methods previously given.

2.54 MOISTURE-STRENGTH RELATIONS FOR PLYWOOD.

2.540 General. The design values given in the plywood strength-property tables 2-13 and 2-14 were calculated from the strength properties of solid wood as given in table 2-6 based on a moisture content of 15 percent; and are applicable for design of aircraft to be used in the continental United States. For design of aircraft to be used

Table 2-12. Approximate methods for calculating the strength and stiffness of plywood ¹

Property	Direction of stress with respect to direction of face grain	Portion of cross-sectional area to be considered	Allowables expressed in proportion of strength values given in table 2-6
Ultimate tensile	Parallel ($F_{t\parallel}$) or perpendicular ($F_{t\perp}$) $\pm 45^\circ$	Parallel plies ² only Full cross-sectional area	Modulus of rupture. One-fourth modulus of rupture.
Ultimate compressive	Parallel ($F_{c\parallel}$) or perpendicular ($F_{c\perp}$) $\pm 45^\circ$ (F_{c45°)	Parallel plies ² only Full cross-sectional area	Maximum crushing strength or fiber stress at proportional limit, as required. One third maximum crushing strength or one-third fiber stress at proportional limit, as required.
Shear	Parallel or perpendicular (F_{sv}) $\pm 45^\circ$	Full cross-sectional area Full cross-sectional area	1.18 times the shearing-strength parallel to grain. 2.35 times the shearing strength parallel to grain
Shear in plane of plies	Parallel, perpendicular, or $\pm 45^\circ$	Joints between ribs, spars, etc., and continuous stressed plywood coverings; joints between webs (plywood) and flanges of I and box-beams; joints between ribs, spars, etc., and stressed plywood panels when plywood terminates at joint--use shear area over support.	One-third the shearing strength parallel to grain for the weakest species.
Load in bending	Parallel or perpendicular	Bending moment $M=KfI/c'$ where I =moment of inertia computed on basis of parallel plies only; c' =distance from neutral axis to outer fiber of outermost ply having its grain in direction of span; $K=1.50$ for three-ply plywood having grain of outer plies perpendicular to span. $K=0.85$ for all other plywood.	Modulus of rupture or fiber stress at proportional limit as required.
Deflection in bending	Parallel or perpendicular	Deflection may be calculated by the usual formulas, taking as the moment of inertia that of the parallel plies plus 1/20 that of the perpendicular plies. (When face plies are parallel, the calculation may be simplified, with but little error, by taking the moment of inertia as that of the parallel plies only).	Modulus of elasticity.
Deformation in tension or compression.	Parallel or perpendicular	Parallel plies ² only	Modulus of elasticity.
Bearing at right angles to plane of plywood.		Loaded area	Compression perpendicular to grain.

¹ These simplified strength calculations are to be used only as a rough guide in preliminary design work, and are not acceptable for final design when the results obtained differ considerably from those obtained by the more accurate methods given in this bulletin.

² By "parallel plies" is meant those plies whose grain direction is parallel to the direction of principal stress.

in tropical conditions, for which 20 percent moisture content is recommended, the design values for plywood may be calculated from basic strength properties of wood at 20 percent moisture content as given in table 2-7, or approximate adjustments may be made as indicated in the following sections.

Adjustment factors by which strength properties of solid wood may be corrected for moisture content are shown in table 2-2. For plywood, moisture corrections are dependent on many variable factors, such as grain direction, combinations of species, and relative thicknesses of plies in each direction, so that any rational method of correction is quite laborious. An approximate method for making moisture corrections to plywood is given in the succeeding sections.

2.541. *Approximate methods for making moisture corrections for plywood strength properties.* A limited number of compression, bending, and shear tests of spruce and Douglas-fir plywood of a few constructions at moisture content values ranging approximately from 6 to 15 percent has indicated that use of the following simplified methods of correcting plywood strength properties will be satisfactory (ref. 2-10 and 2-14).

2.5410. *Moisture corrections for plywood compressive strength (0° or 90° to face grain direction).* Moisture adjustments to the compressive strength of plywood, either parallel or perpendicular to the face grain direction, may be made by direct use of the correction constants given in column (6) of table 2-2.

When more than one species is used in the plywood, the correction constant should be taken for that species having its grain direction parallel to the applied load.

When plies of two species have their grain direction parallel to the applied load, the plywood correction constant should be determined by taking the mean value of the correction constants for the two species based on the relative areas of the longitudinal plies of each.

2.5411. *Moisture correction for plywood tensile strength (0° or 90° to face grain direction).* Data on the effect of moisture on the tensile strength of plywood are lacking. Limited data indicate that the effect on the tensile strength of wood is about one-third as great as on modulus of rupture. The suggested procedure for adjusting the tensile strength of plywood is to follow that for compressive strength as given in the preceding section, using one-third of the correction

factors given for modulus of rupture in column 3 of table 2-2.

2.5412. *Moisture corrections for plywood shear strength (0° or 90° to face grain direction).* Moisture adjustments to the shear strength of plywood, either parallel or perpendicular to the face grain direction, F_{swz} , may be made by direct use of empirical correction constants equal to those given in column (8) of table 2-2 for shear. The use of such moisture adjustment to the shear strength of plywood is not applicable when a moisture content of less than 7 percent is involved.

When more than one species is used, the correction constant should be determined on the basis of the relative areas of each species, considering all plies.

2.5413. *Moisture corrections for plywood compressive strength (any angle to face grain direction).* The compression strength of plywood at any moisture content, and at any angle to the face grain direction, may be found by use of equation 2:51 after first correcting the compression terms F_{cuw} and F_{cuz} in accordance with section 2.5410.

2.5414. *Moisture corrections for plywood tensile strength (any angle to face grain direction).* The tension strength of plywood at any moisture content, and at any angle to the face grain direction, may be found by use of equation 2:53 after first correcting the tension terms F_{tuw} and F_{tuz} in accordance with section 2.5411, and the shear term F_{swz} in accordance with section 2.5412.

2.5415. *Moisture corrections for plywood shear strength (any angle to face grain direction).* The shear strength of plywood at any moisture content, and at any angle to the face grain direction, may be found by use of equations 2:55 or 2:56 after first correcting the various terms in these equations by the methods outlined in the foregoing sections.

2.55. **SPECIFIC GRAVITY-STRENGTH RELATIONS FOR PLYWOOD.** As in solid wood, the strength and elastic properties of plywood increase with an increase in specific gravity. The magnitude of this strength increase, however, cannot be determined by the same convenient exponential equation given in table 2-1.

Specification AN-P-69a, *Plywood and Veneer, Aircraft Flat Panel*, controls the minimum specific gravity of the individual veneers used in the manufacture of the plywood, consequently assuring a minimum final specific gravity of the plywood. The "weight per square foot" column in table 2-13 for various plywood constructions has

Table 2-13. Strength and elastic properties of plywood at 15 percent moisture content
(Based on design data of table 2-6)

THREE-PLY

Nominal thickness	Species 1	Veneer thickness 2	Plywood weight (15 percent moisture) 3	Static bending						Compression						Ultimate strength in tension			Ultimate strength in shear										
				Modulus of elasticity			Moment for fiber stress at proportional limit			Moment for modulus of rupture			Modulus of elasticity 4			Fiber stress at proportional limit			Maximum crushing strength			Ultimate strength in tension							
				E _t /w	E _s	E _s /E _t	Parallel 3	Perpendicular 3	In-lb. per in. of width	In-lb. per in. of width	Parallel 3	Perpendicular 3	In-lb. per in. of width	In-lb. per in. of width	P. s. i.	P. s. i.	P. s. i.	E _{tw}	E _{sz}	F _{rw}	F _{rsz}	F _{crw}	F _{crsz}	F _{tw}	F _{tsz}	F _{tw}	F _{tsz}	F _{tw}	F _{tsz}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)								
In.				1,000	1,000	1,000	In-lb. per in. of width	In-lb. per in. of width	In-lb. per in. of width	In-lb. per in. of width	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.		
0.035	Birch-birch	F&B C	0.146	1,785	149	110	1.27	0.23	2.53	0.562	1,357	739	3,130	1,670	4,430	2,350	10,070	5,030	2,840	2,300	3,320								
.070	Birch-birch	F&B C	.282	1,711	222	186	4.88	1.526	9.69	3.031	1,202	924	2,710	2,090	3,840	2,950	8,630	6,470	2,490	2,790	3,300								
.070	Birch-yellow-poplar	F&B C	.242	1,709	175	137	4.87	.995	9.08	1.890	1,188	642	2,680	1,340	3,800	1,910	8,630	3,600	2,320	1,860	2,650								
.070	Mahogany-mahogany	F&B C	.218	1,271	215	162	4.38	1.557	7.54	2.679	916	719	2,630	2,070	3,770	2,960	6,690	5,010	2,170	2,710	3,010								
.070	Mahogany-yellow-poplar	F&B C	.205	1,266	206	152	4.36	1.078	7.51	2.015	886	662	2,550	1,390	3,650	1,980	6,690	3,690	2,140	1,900	2,590								
.070	Yellow-poplar-yellow-poplar	F&B C	.198	1,134	170	146	2.90	1.015	5.42	1.897	770	661	1,410	1,390	2,300	1,980	4,650	3,950	2,090	1,820	1,990								
.070	Sweetgum-sweetgum	F&B C	.223	1,240	167	137	3.98	1.267	7.19	2.299	874	674	2,090	1,550	2,850	2,190	6,400	4,800	2,230	2,070	2,450								
.070	Sweetgum-yellow-poplar	F&B C	.208	1,239	160	130	3.98	.952	7.19	1.781	868	631	1,900	1,320	2,830	1,880	6,400	3,690	2,170	1,800	2,250								
.070	Douglas-fir-Douglas-fir	F&B C	.223	1,371	209	163	3.80	1.282	7.01	2.369	978	761	2,530	1,970	3,360	2,620	6,230	4,670	2,110	2,420	2,730								
.100	Birch-birch	F&B C	.302	1,737	196	159	10.11	2.838	20.08	5.638	1,258	869	2,840	1,960	4,020	2,780	9,060	6,040	2,020	2,660	3,350								
.100	Birch-yellow-poplar	F&B C	.338	1,735	158	120	10.10	1.876	20.06	3.508	1,215	695	2,810	1,270	3,980	1,800	9,060	3,440	1,860	1,770	2,690								
.100	Mahogany-mahogany	F&B C	.300	1,290	197	143	9.07	2.962	15.60	5.096	955	680	2,750	1,950	3,930	2,800	7,020	4,680	1,700	2,600	3,010								
.100	Mahogany-yellow-poplar	F&B C	.283	1,286	189	135	9.04	2.058	15.56	3.847	928	627	2,670	1,310	3,820	1,870	7,020	3,440	1,670	1,810	2,560								
.100	Yellow-poplar-yellow-poplar	F&B C	.275	1,149	155	131	5.99	1.938	11.20	3.622	797	637	1,670	1,330	2,380	1,900	4,820	3,780	1,620	1,770	2,010								
.100	Sweetgum-sweetgum	F&B C	.303	1,259	148	118	8.25	2.366	14.90	4.274	914	634	2,100	1,450	2,930	2,060	6,720	4,480	1,760	1,970	2,480								
.100	Sweetgum-yellow-poplar	F&B C	.288	1,258	143	112	8.24	1.782	14.89	3.332	908	594	2,030	1,240	2,960	1,770	6,720	3,440	1,700	1,710	2,240								
.100	Douglas-fir-Douglas-fir	F&B C	.308	1,392	189	143	7.86	2.419	14.52	4.470	1,021	718	2,650	1,860	3,510	2,470	6,540	4,360	1,630	2,310	2,730								

.125	Birch-birch.....	F&B C	.034 .060	.494	1,668	205	229	15.17	5,544	30.14	11,015	1,124	1,063	2,540	2,260	3,590	3,200	8,020	7,050	1,800	2,970	3,200
.125	Birch-yellow-poplar.....	F&B C	.034 .060	.414	1,665	203	166	15.14	3,560	30.08	6,655	1,100	693	2,500	1,450	3,540	2,070	8,020	4,030	1,610	2,000	2,660
.125	Mahogany-mahogany.....	F&B C	.034 .060	.377	1,241	246	193	13.63	5,508	23.46	9,478	861	774	2,470	2,240	3,540	3,190	6,220	5,440	1,440	2,840	2,970
.125	Mahogany-yellow-poplar.....	F&B C	.034 .060	.352	1,234	213	181	13.56	3,798	23.34	7,100	828	713	2,340	1,490	3,410	2,120	6,220	4,030	1,440	2,010	2,660
.125	Yellow-poplar-yellow-poplar.....	F&B C	.034 .060	.342	1,100	203	180	8.96	3,693	16.76	6,904	717	717	1,590	1,500	2,140	2,140	4,300	4,300	1,400	1,920	1,920
.125	Sweetgum-sweetgum.....	F&B C	.034 .060	.388	1,209	198	169	12.38	4,588	22.37	8,287	818	730	1,870	1,670	2,600	2,350	5,950	5,250	1,540	2,210	2,370
.125	Sweetgum-yellow-poplar.....	F&B C	.034 .060	.358	1,208	180	159	12.37	3,438	22.34	6,428	811	684	1,860	1,430	2,640	2,040	5,950	4,630	1,470	1,930	2,210
.125	Douglas-fir-Douglas-fir.....	F&B C	.034 .060	.388	1,338	213	198	11.81	4,582	21.81	8,406	917	822	2,380	2,140	3,150	2,830	5,790	5,110	1,410	2,540	2,680
.155	Birch-birch.....	F&B C	.040 .080	.612	1,629	304	268	22.78	9,426	45.26	18,726	1,063	1,063	2,400	2,400	3,400	3,400	7,550	7,550	1,130	3,100	3,100
.155	Birch-yellow-poplar.....	F&B C	.040 .080	.505	1,625	229	193	22.73	5,992	45.16	11,203	1,047	733	2,360	1,510	3,340	2,190	7,550	4,300	1,430	2,100	2,640
.155	Mahogany-mahogany.....	F&B C	.040 .080	.465	1,213	273	222	20.50	9,206	35.27	15,839	818	818	2,350	2,350	3,360	3,360	5,850	5,850	1,310	2,910	2,910
.155	Mahogany-yellow-poplar.....	F&B C	.040 .080	.432	1,205	258	207	20.36	6,329	35.04	11,883	783	752	2,250	1,570	3,220	2,240	5,850	4,360	1,270	2,040	2,580
.155	Yellow-poplar-yellow-poplar.....	F&B C	.040 .080	.398	1,100	203	180	13.78	5,678	25.77	10,616	717	717	1,500	1,500	2,140	2,140	4,300	4,300	1,230	1,920	1,920
.155	Sweetgum-sweetgum.....	F&B C	.040 .080	.478	1,181	226	197	18.60	7,781	33.59	14,056	774	774	1,770	1,770	2,520	2,520	5,600	5,600	1,370	2,300	2,360
.155	Sweetgum-yellow-poplar.....	F&B C	.040 .080	.438	1,179	215	186	18.57	5,821	33.55	10,883	767	724	1,760	1,520	2,500	2,160	5,600	4,360	1,360	2,620	2,160
.155	Douglas-fir-Douglas-fir.....	F&B C	.040 .080	.478	1,308	273	229	17.74	7,769	32.78	14,242	869	869	2,250	2,250	2,960	2,960	5,450	5,450	1,240	2,620	2,620
.185	Birch-birch.....	F&B C	.047 .095	.718	1,626	308	272	32.38	13,542	64.33	26,906	1,058	1,068	2,390	2,410	3,380	3,410	7,510	7,590	1,510	3,100	3,090
.185	Birch-yellow-poplar.....	F&B C	.047 .095	.591	1,622	231	195	32.31	8,602	64.19	16,082	1,042	737	2,350	1,540	3,330	2,200	7,510	4,320	1,310	2,110	2,640
.185	Mahogany-mahogany.....	F&B C	.047 .095	.545	1,211	276	225	29.14	13,208	50.14	22,725	814	821	2,340	2,360	3,350	3,380	5,820	5,880	1,190	2,920	2,910
.185	Mahogany-yellow-poplar.....	F&B C	.047 .095	.505	1,203	260	209	28.95	9,078	49.81	16,973	779	755	2,240	1,580	3,210	2,250	5,820	4,320	1,150	2,100	2,580
.185	Yellow-poplar-yellow-poplar.....	F&B C	.047 .095	.466	1,098	206	183	19.59	8,158	36.63	15,252	714	721	1,490	1,510	2,130	2,150	4,280	4,320	1,120	1,920	1,910
.185	Sweetgum-sweetgum.....	F&B C	.047 .095	.560	1,178	229	200	26.43	11,176	47.75	20,180	770	778	1,770	1,780	2,510	2,530	5,570	5,630	1,250	2,360	2,290
.185	Sweetgum-yellow-poplar.....	F&B C	.047 .095	.513	1,177	217	188	26.40	8,360	47.68	15,630	763	728	1,750	1,520	2,480	2,170	5,570	4,320	1,180	2,020	2,150
.185	Douglas-fir-Douglas-fir.....	F&B C	.047 .095	.560	1,305	276	232	25.22	11,066	46.59	20,444	805	873	2,240	2,260	2,980	3,000	5,420	5,480	1,130	2,620	2,620

See footnotes at end of table.

Table 2-13. Strength and elastic properties of plywood at 15 percent moisture content ---Continued

APPENDIX

Nominal thickness	Species	Veneer thickness ²	Plywood weight (lb per sq ft)	Static bending						Compression						Ultimate strength in tension		Ultimate strength in shear			
				Modulus of elasticity		Moment for fiber stress at proportional limit		Moment for modulus of rupture		Modulus of elasticity		Fiber stress at proportional limit		Maximum crushing strength							
				$E_{1/2}$	$E_{1/4}$	Parallel ³	Perpendicular ³	Parallel ³	Perpendicular ³	Parallel ³	Perpendicular ³	E_x	E_y	F_{100}	F_{150}	F_{180}	F_{200}	F_{250}	F_{300}		
																				(5)	(6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
.160	Birch-birch	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Birch-yellow-poplar	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Mahogany-mahogany	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Mahogany-yellow-poplar	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Yellow-poplar-yellow-poplar	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Sweetgum-sweetgum	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Sweetgum-yellow-poplar	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Douglas-fir-Douglas-fir	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Birch-birch	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Birch-yellow-poplar	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Mahogany-mahogany	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	
.160	Mahogany-yellow-poplar	.030	.030	1,000	1,000	1,000	10.46	10.46	42.32	13.26	1,050	638	2,220	1,340	3,160	1,900	3,700	2,050	1,840	2,400	

.190	Yellow-poplar-yellow-poplar	F&B XB C	.034 .047 .034	.507	.923	381	301	17.37	10.84	32.48	20.26	744	690	1,560	1,450	2,220	2,060	4,480	4,120	1,680	1,870	1,950
.190	Sweetgum-sweetgum	F&B XB C	.034 .047 .030	.594	1,002	405	351	23.71	14.47	42.83	26.14	788	759	1,810	1,740	2,570	2,470	5,720	5,480	1,820	2,270	2,330
.190	Sweetgum-yellow-poplar	F&B XB C	.034 .017 .034	.541	988	390	306	23.38	10.93	42.23	20.44	778	695	1,630	1,460	2,320	2,070	5,380	4,120	1,730	1,930	2,020
.190	Douglas-fir-Douglas-fir	F&B XB C	.034 .031 .047 .030	.594	1,114	467	430	22.71	13.90	41.95	25.85	885	854	2,290	2,210	3,040	2,940	5,500	5,340	1,090	2,600	2,640
.225	Birch-birch	F&B XB C	.040 .030 .030	.893	1,364	569	539	40.18	25.37	79.84	50.41	1,021	1,106	2,300	2,490	3,260	3,530	7,220	7,880	1,890	3,170	3,010
.225	Birch-yellow-poplar	F&B XB C	.040 .060 .030	.693	1,354	404	374	39.90	15.64	79.28	29.24	918	757	1,920	1,590	2,740	2,260	6,370	4,490	1,640	2,130	2,340
.225	Mahogany-mahogany	F&B XB C	.040 .030 .030	.682	1,025	461	419	36.49	23.48	62.78	40.40	788	848	2,260	2,440	3,210	3,490	5,600	6,100	1,570	2,960	2,860
.225	Mahogany-yellow-poplar	F&B XB C	.040 .060 .030	.629	1,008	428	386	35.87	15.08	61.71	29.87	734	770	1,540	1,610	2,190	2,200	5,190	4,490	1,530	2,100	1,970
.225	Yellow-poplar-yellow-poplar	F&B XB C	.040 .060 .034	.596	913	391	372	24.09	15.54	45.03	29.06	709	734	1,470	1,540	2,000	2,190	4,190	4,410	1,500	1,940	1,890
.225	Sweetgum-sweetgum	F&B XB C	.010 .030 .030	.702	990	417	393	32.84	20.79	59.33	37.56	743	804	1,700	1,840	2,420	2,620	5,360	5,840	1,640	2,350	2,240
.225	Sweetgum-yellow-poplar	F&B XB C	.010 .030 .030	.627	986	392	368	32.71	15.47	59.10	28.92	723	751	1,510	1,570	2,160	2,240	5,620	4,490	1,550	2,050	1,950
.225	Douglas-fir-Douglas-fir	F&B XB C	.010 .030 .030	.702	1,100	480	444	31.46	20.09	58.13	37.12	836	902	2,170	2,340	2,880	3,100	5,210	5,690	1,510	2,600	2,570
.250	Birch-birch	F&B XB C	.047 .030 .010	.981	1,415	518	487	51.48	29.18	102.28	57.97	1,117	1,010	2,520	2,280	3,570	3,230	7,970	7,130	1,800	2,990	3,190
.250	Birch-yellow-poplar	F&B XB C	.017 .060 .017	.784	1,384	382	351	50.43	18.39	100.20	34.38	1,008	676	2,110	1,420	3,000	2,020	6,990	3,950	1,540	1,910	2,390
.250	Mahogany-mahogany	F&B XB C	.017 .030 .010	.748	1,062	425	381	46.65	27.13	80.27	46.07	856	780	2,400	2,240	3,520	3,210	6,170	5,540	1,450	2,850	2,970
.250	Mahogany-yellow-poplar	F&B XB C	.017 .060 .010	.698	1,032	406	363	45.36	18.80	78.05	35.16	818	689	1,710	1,440	2,440	2,050	5,750	3,950	1,440	1,940	2,080
.250	Yellow-poplar-yellow-poplar	F&B XB C	.017 .030 .047	.659	914	360	340	39.75	18.01	57.48	33.67	770	664	1,610	1,300	2,300	1,980	4,650	3,050	1,400	1,820	1,990
.250	Sweetgum-sweetgum	F&B XB C	.017 .030 .040	.770	1,027	380	355	42.06	23.92	75.99	43.21	812	735	1,860	1,690	2,650	2,300	5,910	5,290	1,510	2,220	2,370

See footnotes at end of table.

Table 2-13. Strength and elastic properties of plywood at 15 percent moisture content—Continued

FIVE PLY—Continued

Nominal thickness	Species 1	Veneer thickness 2	Plywood weight (15 percent moisture) 2	Static bending						Compression				Ultimate strength in tension		Ultimate strength in shear					
				Modulus of elasticity		Moment for fiber stress at proportional limit		Moment for modulus of rupture		Modulus of elasticity 4		Fiber stress at proportional limit		Maximum crushing strength	F _{tuw}	F _{tuz}	F _{usr}	F _{usr}			
				E _{1w}	E _{1z}	Parallel 3	Perpendicular 3	Parallel 3	Perpendicular 3	E _w	E _z	F _{cpw}	F _{cpz}								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
In. 0.250	Sweetgum-yellow-poplar	In.	P. s. i.	1,000	1,000	1,000	In.-lb. per in. of width	In.-lb. per in. of width	In.-lb. per in. of width	In.-lb. per in. of width	1,000	1,000	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.	P. s. i.
		0.047	0 706	370	345	41.38	18.18	74.75	33.99	806	809	1,600	1,400	2,400	2,000	5,580	3,950	1,450	1,890	2,050	
		XB																			
.250	Douglas-fir-Douglas-fir	In.	P. s. i.	1,141	440	402	40.26	23.16	74.38	42.80	911	928	2,360	2,150	3,130	2,850	5,750	5,150	1,410	2,550	2,670
		0.047																			
		XB																			
.315	Birch-birch	In.	P. s. i.	1,422	511	480	82.12	46.04	103.17	91.48	1,053	1,063	2,400	2,400	3,400	3,400	7,550	7,550	1,620	3,100	3,100
		0.060																			
		XB																			
.315	Birch-yellow-poplar	In.	P. s. i.	1,398	376	344	80.72	28.99	100.39	54.19	973	715	2,040	1,500	2,900	2,130	6,780	4,210	1,370	2,030	2,300
		0.060																			
		XB																			
.315	Mahogany-mahogany	In.	P. s. i.	1,066	420	376	74.40	42.87	128.02	73.76	818	818	2,350	2,350	3,360	3,360	5,850	5,850	1,200	2,910	2,910
		0.047																			
		XB																			
.315	Mahogany-yellow-poplar	In.	P. s. i.	1,040	400	356	72.54	29.68	124.81	55.49	780	728	1,630	1,530	2,320	2,170	5,530	4,210	1,250	2,020	2,030
		0.060																			
		XB																			
.315	Yellow-poplar-yellow-poplar	In.	P. s. i.	950	354	334	49.15	28.35	91.89	53.00	731	703	1,530	1,470	2,180	2,100	4,300	4,210	1,220	1,890	1,940
		0.060																			
		XB																			
.315	Sweetgum-sweetgum	In.	P. s. i.	1,032	375	350	67.10	37.76	121.21	68.20	774	774	1,770	1,770	2,520	2,520	5,000	5,600	1,300	2,300	2,300
		0.047																			
		XB																			
.315	Sweetgum-yellow-poplar	In.	P. s. i.	1,017	364	339	66.16	28.64	119.52	53.54	767	708	1,610	1,480	2,290	2,110	5,350	4,210	1,270	1,960	2,010
		0.060																			
		XB																			
.315	Douglas-fir-Douglas-fir	In.	P. s. i.	1,146	435	397	64.22	36.59	118.64	67.59	869	869	2,250	2,250	2,990	2,990	5,450	5,450	1,230	2,620	2,620
		0.060																			
		XB																			
.375	Birch-birch	In.	P. s. i.	1,279	654	626	104.68	77.22	207.97	153.43	1,088	1,088	2,430	2,340	3,480	3,320	7,710	7,360	1,500	3,050	3,150
		0.060																			
		XB																			
.375	Birch-yellow-poplar	In.	P. s. i.	1,204	460	432	103.47	47.38	205.58	88.58	937	710	1,900	1,490	2,790	2,120	6,410	4,190	1,230	2,010	2,320
		0.060																			
		XB																			

.375	Mahogany-mahogany	F&B XB C	.060 .065 .080	1.122	.945	522	482	95.41	70.94	104.16	122.06	835	800	2,400	2,300	3,440	3,290	6,000	5,700	1,180	2,880	2,950
.375	Mahogany-yellow-poplar	F&B XB C	.060 .065 .080	1.010	.944	482	443	93.30	48.15	160.54	90.02	775	721	1,620	1,510	2,310	2,150	5,360	4,190	1,130	2,000	2,020
.375	Yellow-poplar-yellow-poplar	F&B XB C	.060 .065 .080	.900	.863	440	422	63.30	46.68	118.34	87.27	734	700	1,540	1,470	2,190	2,080	4,410	4,190	1,110	1,890	1,940
.375	Sweetgum-sweetgum	F&B XB C	.060 .065 .080	1.155	.929	478	456	85.59	63.21	154.62	114.19	792	756	1,820	1,730	2,580	2,460	5,740	5,460	1,240	2,260	2,330
.375	Sweetgum-yellow-poplar	F&B XB C	.060 .065 .080	1.020	.924	449	427	85.13	47.00	153.78	87.86	764	705	1,600	1,480	2,280	2,100	5,210	4,190	1,150	1,950	2,000
.375	Douglas-fir-Douglas-fir	F&B XB C	.060 .065 .080	1.155	1.034	546	513	82.14	60.88	151.74	112.47	889	850	2,300	2,200	3,060	2,920	5,590	5,310	1,120	2,590	2,650

SEVEN-PLY (All plies of equal thickness)

0.410	Birch-birch		0.060	1.02	1.340	594	567	131.1	80.91	280.5	160.75	1,202	924	2,710	2,000	3,840	2,950	8,630	6,470	1,720	2,790	3,300
.410	Birch-yellow-poplar		.060	1.22	1.286	418	391	125.8	40.60	250.0	92.84	1,000	632	2,090	1,320	2,980	1,890	6,770	3,690	1,440	1,820	2,320
.410	Mahogany-mahogany		.060	1.23	1.008	478	441	110.2	75.00	205.0	120.20	916	719	2,630	2,070	3,770	2,960	6,690	5,010	1,400	2,710	3,010
.410	Mahogany-yellow-poplar		.060	1.10	.981	439	401	116.0	50.51	190.5	94.44	849	643	1,780	1,350	2,530	1,920	5,800	3,690	1,350	1,820	2,060
.410	Yellow-poplar-yellow-poplar		.060	1.06	.905	399	382	79.3	48.88	148.2	91.38	811	623	1,700	1,310	2,420	1,800	4,910	3,690	1,330	1,740	2,020
.410	Sweetgum-sweetgum		.060	1.26	.973	434	413	107.2	66.31	183.6	119.79	874	674	2,000	1,550	2,850	2,100	6,400	4,800	1,460	2,070	2,450
.410	Sweetgum-yellow-poplar		.060	1.12	.962	407	386	106.0	49.23	191.5	92.04	839	627	1,760	1,310	2,500	1,870	5,600	3,690	1,370	1,780	2,070
.410	Douglas-fir-Douglas-fir		.060	1.26	1.082	499	466	102.7	64.16	189.7	118.54	978	761	2,530	1,970	3,360	2,620	6,230	4,670	1,340	2,420	2,730
.460	Birch-birch		.068	1.82	1.340	593	567	165.0	101.84	327.9	202.35	1,202	924	2,710	2,090	3,840	2,950	8,630	6,470	1,630	2,790	3,300
.460	Birch-yellow-poplar		.068	1.37	1.286	418	391	158.4	62.51	314.7	116.87	1,000	632	2,090	1,320	2,980	1,890	6,770	3,690	1,350	1,820	2,320
.460	Mahogany-mahogany		.068	1.38	1.008	478	441	150.0	94.52	238.1	162.63	916	719	2,630	2,070	3,770	2,960	6,690	5,010	1,310	2,710	3,010
.460	Mahogany-yellow-poplar		.068	1.24	.981	439	401	146.0	63.58	251.1	118.87	849	643	1,780	1,350	2,530	1,920	5,800	3,690	1,260	1,820	2,090
.460	Yellow-poplar-yellow-poplar		.068	1.19	.905	399	382	99.8	61.53	186.6	115.03	811	623	1,700	1,310	2,420	1,800	4,910	3,690	1,240	1,740	2,020
.460	Sweetgum-sweetgum		.068	1.42	.973	434	413	134.9	83.47	243.7	150.79	874	674	2,000	1,550	2,850	2,100	6,400	4,800	1,380	2,070	2,450
.460	Sweetgum-yellow-poplar		.068	1.25	.962	407	386	133.4	61.97	241.0	115.86	839	627	1,760	1,310	2,500	1,870	5,600	3,690	1,280	1,780	2,070
.460	Douglas-fir-Douglas-fir		.068	1.42	1.082	499	466	129.3	80.76	248.9	149.21	978	761	2,530	1,970	3,360	2,620	6,230	4,670	1,250	2,420	2,730
.540	Birch-birch		.080	2.13	1.340	593	567	227.4	140.35	451.8	278.85	1,202	924	2,710	2,090	3,840	2,950	8,630	6,470	1,530	2,790	3,300
.540	Birch-yellow-poplar		.080	1.60	1.286	418	391	218.3	86.14	433.6	161.05	1,000	632	2,090	1,320	2,980	1,890	6,770	3,690	1,250	1,820	2,320
.540	Mahogany-mahogany		.080	1.62	1.008	478	441	206.7	130.25	355.7	224.12	916	719	2,630	2,070	3,770	2,960	6,690	5,010	1,210	2,710	3,010
.540	Mahogany-yellow-poplar		.080	1.45	.981	439	401	201.1	87.62	346.1	163.82	849	643	1,780	1,350	2,530	1,920	5,800	3,690	1,100	1,820	2,090
.540	Yellow-poplar-yellow-poplar		.080	1.38	.905	399	382	137.5	84.79	257.1	158.52	811	623	1,700	1,310	2,420	1,860	4,910	3,690	1,130	1,740	2,020
.540	Sweetgum-sweetgum		.080	1.66	.973	434	413	185.9	115.63	335.8	207.80	874	674	2,000	1,550	2,850	2,190	6,400	4,800	1,270	2,070	2,450
.540	Sweetgum-yellow-poplar		.080	1.46	.962	407	386	183.9	85.40	332.1	150.66	839	627	1,760	1,310	2,500	1,870	5,600	3,690	1,170	1,780	2,070
.540	Douglas-fir-Douglas-fir		.080	1.66	1.082	499	466	178.2	111.30	329.2	205.62	978	761	2,530	1,970	3,360	2,620	6,230	4,670	1,150	2,420	2,730

NINE-PLY (All plies of equal thickness)

0.500	Birch-birch		0.068	2.34	1.250	652	652	255.0	178.0	506.6	353.6	1,171	955	2,640	2,150	3,740	3,050	8,390	6,710	1,660	2,870	3,270
.500	Birch-yellow-poplar		.068	1.89	1.176	619	627	238.3	170.6	473.4	339.0	944	798	1,980	1,670	2,810	2,380	6,220	5,270	1,440	2,220	2,480
.500	Mahogany-mahogany		.068	1.78	.950	536	565	232.6	164.9	400.3	263.7	894	741	2,570	2,130	3,680	3,050	5,500	5,200	1,340	2,700	3,000
.500	Mahogany-yellow-poplar		.068	1.64	.932	523	492	228.1	160.4	392.5	276.0	845	689	1,750	1,440	2,490	2,050	5,470	4,510	1,300	1,920	2,180
.500	Yellow-poplar-yellow-poplar		.068	1.53	.850	454	440	154.2	107.6	288.3	201.0	790	644	1,600	1,360	2,300	1,920	4,780	3,820	1,270	1,790	2,010

See footnotes at end of table.

Table 2-13. Strength and elastic properties of plywood at 15 percent moisture content.—Continued

NINE-PLY (All plies of equal thickness)—Continued

Non- final thick- ness	Species ¹	Veneer thickness ²	Ply- wood- weight (15 per- cent mois- ture) ³	Static bending				Compression				Ultimate strength in tension		Ultimate strength in shear							
				Modulus of elasticity		Moment for fiber stress at proportional limit		Moment for modulus of rupture		Fiber stress at propor- limit		Maximum crushing strength		F_{tuz}	F_{tuz}	F_{suz}	F_{ssz}				
				E_{fw}	E'_{fs}	Paral- lel ³	Per- pen- dicu- lar ³	In.-lb. per in. of width	In.-lb. per in. of width	In.-lb. per in. of width	E_x	E_y	F_{cpw}					F_{spz}	F_{cpw}	F_{csz}	
																					(5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
$In.$		$In.$	$P. s. i.$	$1,000$	$P. s. i.$	$1,000$	$In.-lb.$ per in. of width	$In.-lb.$ per in. of width	$In.-lb.$ per in. of width	$In.-lb.$ per in. of width	$p. s. i.$	$p. s. i.$	$P. s. i.$	$P. s. i.$	$P. s. i.$	$P. s. i.$	$P. s. i.$	$P. s. i.$	$P. s. i.$	$P. s. i.$	$P. s. i.$
.500	Sweetgum-sweetgum	0.068	1.83	914	493	475	208.5	145.8	376.7	263.4	882	696	1,950	1,400	2,770	2,270	6,220	4,980	1,400	2,420	
.500	Sweetgum-yellow-poplar	0.068	1.66	901	488	470	205.6	144.2	371.5	260.5	815	669	1,710	1,400	2,430	2,000	5,360	4,400	1,330	2,130	
.500	Douglas-fir-Douglas-fir	0.068	1.83	1,018	562	535	200.2	141.0	369.8	260.4	954	785	2,470	2,030	3,280	2,700	6,060	4,840	1,280	2,470	
.695	Birch-birch	0.080	2.74	1,259	675	652	353.8	246.9	703.0	490.6	1,171	955	2,640	2,150	3,740	3,050	8,390	6,710	1,550	2,870	
.695	Birch-yellow-poplar	0.080	2.21	1,176	649	627	330.6	236.8	656.9	470.4	944	798	1,980	1,670	2,810	2,380	6,220	5,270	1,330	2,220	
.695	Mahogany-mahogany	0.080	2.08	950	536	505	322.8	228.8	555.4	393.6	894	741	2,570	2,130	3,680	3,050	6,500	5,290	1,230	2,460	
.695	Mahogany-yellow-poplar	0.080	1.91	932	523	492	316.5	222.6	544.6	383.0	835	689	1,750	1,410	2,490	2,050	5,470	4,510	1,190	2,760	
.695	Yellow-poplar-yellow-poplar	0.080	1.78	850	454	440	213.9	149.2	490.0	278.9	790	644	1,660	1,350	2,360	1,920	4,780	3,820	1,160	2,180	
.695	Sweetgum-sweetgum	0.080	2.14	914	493	475	289.4	202.3	522.7	365.5	852	696	1,950	1,600	2,770	2,270	6,220	4,980	1,200	2,420	
.695	Sweetgum-yellow-poplar	0.080	1.94	901	488	470	285.3	200.1	515.4	361.5	815	669	1,710	1,400	2,430	2,000	5,360	4,400	1,220	2,130	
.695	Douglas-fir-Douglas-fir	0.080	2.14	1,018	562	535	277.8	195.6	513.2	361.4	954	785	2,470	2,030	3,280	2,700	6,060	4,840	1,170	2,470	

ELEVEN-PLY (all plies of equal thickness)

0.850	Birch-birch	0.080	3.35	1,206	727	708	507.2	382.0	1,007.7	750.0	1,152	975	2,600	2,200	3,680	3,130	8,240	6,800	1,560	2,910	3,240
0.850	Birch-yellow-poplar	0.080	2.60	1,094	677	658	400.0	354.2	914.0	703.8	903	783	1,890	1,610	2,690	2,340	6,870	5,080	1,310	2,170	2,380
0.850	Mahogany-mahogany	0.080	2.54	913	573	547	463.9	353.8	798.3	608.7	880	755	2,530	2,170	3,620	3,110	6,380	5,320	1,240	2,800	2,990
0.850	Mahogany-yellow-poplar	0.080	2.31	887	553	526	450.4	340.1	775.0	585.1	813	694	1,700	1,450	2,430	2,070	4,470	3,910	1,200	1,930	2,130
0.850	Yellow-poplar-yellow-poplar	0.080	2.18	814	490	477	306.6	230.8	573.3	431.5	777	657	1,630	1,380	2,320	1,960	4,690	3,910	1,170	1,810	2,000
0.850	Sweetgum-sweetgum	0.080	2.62	876	531	516	414.0	313.0	740.5	565.4	837	710	1,920	1,630	2,780	2,310	6,110	5,090	1,310	2,160	2,410
0.850	Sweetgum-yellow-poplar	0.080	2.34	850	522	507	406.6	307.3	734.6	555.2	798	678	1,670	1,420	2,380	2,020	5,190	4,380	1,220	1,880	2,090
0.850	Douglas-fir-Douglas-fir	0.080	2.62	977	603	580	398.8	302.5	736.8	558.9	938	800	2,430	2,070	3,230	2,750	6,950	4,950	1,180	2,500	2,710
1.010	Birch-birch	0.095	3.96	1,206	727	708	716.1	530.4	1,422.8	1,071.6	1,152	975	2,600	2,200	3,680	3,130	8,240	6,800	1,460	2,910	3,240
1.010	Birch-yellow-poplar	0.095	3.07	1,094	677	658	649.5	500.2	1,290.5	903.7	903	783	1,890	1,610	2,690	2,340	5,870	5,090	1,210	2,170	2,380
1.010	Mahogany-mahogany	0.095	3.00	913	573	547	655.1	490.5	1,127.1	850.4	880	755	2,530	2,170	3,620	3,110	6,380	5,320	1,140	2,800	2,990
1.010	Mahogany-yellow-poplar	0.095	2.72	887	553	526	635.9	480.1	1,094.2	826.1	813	694	1,700	1,450	2,430	2,070	4,470	3,910	1,090	1,930	2,130
1.010	Yellow-poplar-yellow-poplar	0.095	2.56	814	490	477	432.9	325.9	860.4	609.2	777	657	1,630	1,380	2,320	1,960	4,690	3,910	1,070	1,810	2,000
1.010	Sweetgum-sweetgum	0.095	3.09	876	531	516	585.8	441.9	1,088.3	798.3	837	710	1,920	1,630	2,780	2,310	6,110	5,090	1,200	2,160	2,410
1.010	Sweetgum-yellow-poplar	0.095	2.75	850	522	507	574.1	433.9	1,037.1	783.8	798	678	1,670	1,420	2,380	2,020	5,190	4,380	1,120	1,880	2,090
1.010	Douglas-fir-Douglas-fir	0.095	3.00	977	603	580	563.1	427.2	1,040.3	780.2	938	800	2,430	2,070	3,230	2,750	6,950	4,950	1,080	2,500	2,710

¹ Grain direction of adjacent plies at right angles; all veneer rotary cut, except Douglas-fir and mahogany, which are quarter sliced.

² Constructions and veneer thickness taken from Specification AN-P-69A (Plywood and Veneer: Aircraft Flat Panel). In determining the values in this table, the veneer thicknesses were reduced proportionately so that their sums equalled the nominal plywood thickness, except in computing the weight per square foot of panel. A weight of 0.012 pound per square foot of glue line was allowed

In all cases in computing the weight of the plywood. In this column F indicates face; B , back; and C , core.

³ Parallel and perpendicular refer to the relation between the grain direction of the face plies and the direction of the span. In panels, the direction of the span will be determined by the direction of the plywood strip being considered.

⁴ The values of E_x and E_y apply also to tension.

been based on the average specific gravity values for wood listed in table 2-6.

The strength properties for a piece of plywood are merely the composite strengths of each individual veneer in the direction being considered. Therefore, to make a rational specific gravity correction to plywood strength test data, it is first necessary to determine the specific gravity of each individual veneer and then correct its strength properties to correspond to the average specific gravity value given in table 2-6 for that species. To do this, of course, is impractical and the problem is further complicated by the effect of glue impregnation.

When substantiating the strength of a plywood structure, or when establishing design values from static tests, the weight per square foot of the plywood used in the specimens should be near the values given in table 2-13 to minimize the effect of high or low specific gravity values (ref. 2-22).

2.56. STRESS-STRAIN RELATIONS FOR WOOD AND PLYWOOD. When stresses are applied to wood

or plywood in a direction at an angle to the grain, the resulting strains are quite different from those obtained in isotropic materials. The general equations (ref. 2-52 and 2-53) relating the strains to the stresses shown in figure 2-12 are:

$$\begin{aligned} e_1 &= a_{11}f_1 + a_{12}f_2 + a_{13}f_{s12} \\ e_2 &= a_{21}f_1 + a_{22}f_2 + a_{23}f_{s12} \\ e_{s12} &= a_{31}f_1 + a_{32}f_2 + a_{33}f_{s12} \end{aligned} \quad (2:34)$$

in which the stresses and strains are referred to orthogonal axes 1 and 2.

f_1 and e_1 are stress and strain in the direction of axis 1.

f_2 and e_2 are stress and strain in the direction of axis 2.

f_{s12} and e_{s12} are shear stress and shear strain associated with the directions of axes 1 and 2, and in which:

$$a_{11} = \frac{\sin^4 \theta}{E_x} + \frac{\cos^4 \theta}{E_w} + \left[\frac{1}{G_{wx}} - \frac{2\mu_{wx}}{E_w} \right] \sin^2 \theta \cos^2 \theta$$

$$a_{22} = \frac{\sin^4 \theta}{E_w} + \frac{\cos^4 \theta}{E_x} + \left[\frac{1}{G_{wx}} - \frac{2\mu_{wx}}{E_w} \right] \sin^2 \theta \cos^2 \theta$$

$$a_{33} = 4 \left[\frac{1}{E_w} + \frac{1}{E_x} + \frac{2\mu_{wx}}{E_w} \right] \sin^2 \theta \cos^2 \theta + \frac{1}{G_{wx}} (\cos^2 \theta - \sin^2 \theta)^2$$

$$a_{21} = a_{12} = \left[\frac{1}{E_w} + \frac{1}{E_x} - \frac{1}{G_{wx}} \right] \sin^2 \theta \cos^2 \theta - \frac{\mu_{wx}}{E_w} (\sin^4 \theta + \cos^4 \theta)$$

$$a_{31} = a_{13} = \left[\frac{1}{G_{wx}} - \frac{2\mu_{wx}}{E_w} - \frac{2}{E_x} \right] \sin \theta \cos^3 \theta - \left[\frac{1}{G_{wx}} - \frac{2\mu_{wx}}{E_w} - \frac{2}{E_x} \right] \sin^3 \theta \cos \theta$$

$$a_{32} = a_{23} = \left[\frac{1}{G_{wx}} - \frac{2\mu_{wx}}{E_w} - \frac{2}{E_x} \right] \sin^3 \theta \cos \theta - \left[\frac{1}{G_{wx}} - \frac{2\mu_{wx}}{E_w} - \frac{2}{E_x} \right] \sin \theta \cos^3 \theta$$

where θ is measured positively from the direction of the face grain of the plywood to the direction of axis 1, as shown in figure 2-12.

The inverses of equations (2:26) are:

$$f_1 = b_{11}e_1 + b_{12}e_2 + b_{13}e_{s12}$$

$$f_2 = b_{21}e_1 + b_{22}e_2 + b_{23}e_{s12} \quad (2:35)$$

$$f_{s12} = b_{31}e_1 + b_{32}e_2 + b_{33}e_{s12}$$

in which

$$b_{11} = \frac{1}{\lambda} [E_w \cos^4 \theta + E_x \sin^4 \theta + (2E_w \mu_{wx} + 4\lambda G_{wx}) \sin^2 \theta \cos^2 \theta]$$

$$b_{22} = \frac{1}{\lambda} [E_x \cos^4 \theta + E_w \sin^4 \theta + (2E_w \mu_{wx} + 4\lambda G_{wx}) \sin^2 \theta \cos^2 \theta]$$

$$b_{33} = \frac{1}{\lambda} (E_w + E_x - 2E_w \mu_{wx}) \sin^2 \theta \cos^2 \theta + G_{wx} (\cos^2 \theta - \sin^2 \theta)^2$$

$$b_{21}=b_{12}=\frac{1}{\lambda} [(E_u+E_v-4\lambda G_{uv}) \sin^2 \theta \cos^2 \theta + E_u \mu_{vu} (\cos^4 \theta + \sin^4 \theta)]$$

$$b_{31}=b_{13}=\frac{1}{\lambda} [(E_x-E_v \mu_{xv}-2\lambda G_{vx}) \sin^3 \theta \cos \theta - (E_u-E_v \mu_{xu}-2\lambda G_{ux}) \sin \theta \cos^3 \theta]$$

$$b_{32}=b_{23}=\frac{1}{\lambda} [(E_x-E_v \mu_{xv}-2\lambda G_{vx}) \sin \theta \cos^3 \theta - (E_u-E_v \mu_{xu}-2\lambda G_{ux}) \sin^3 \theta \cos \theta]$$

$$\lambda = 1 - \mu_{wx} \mu_{xw}$$

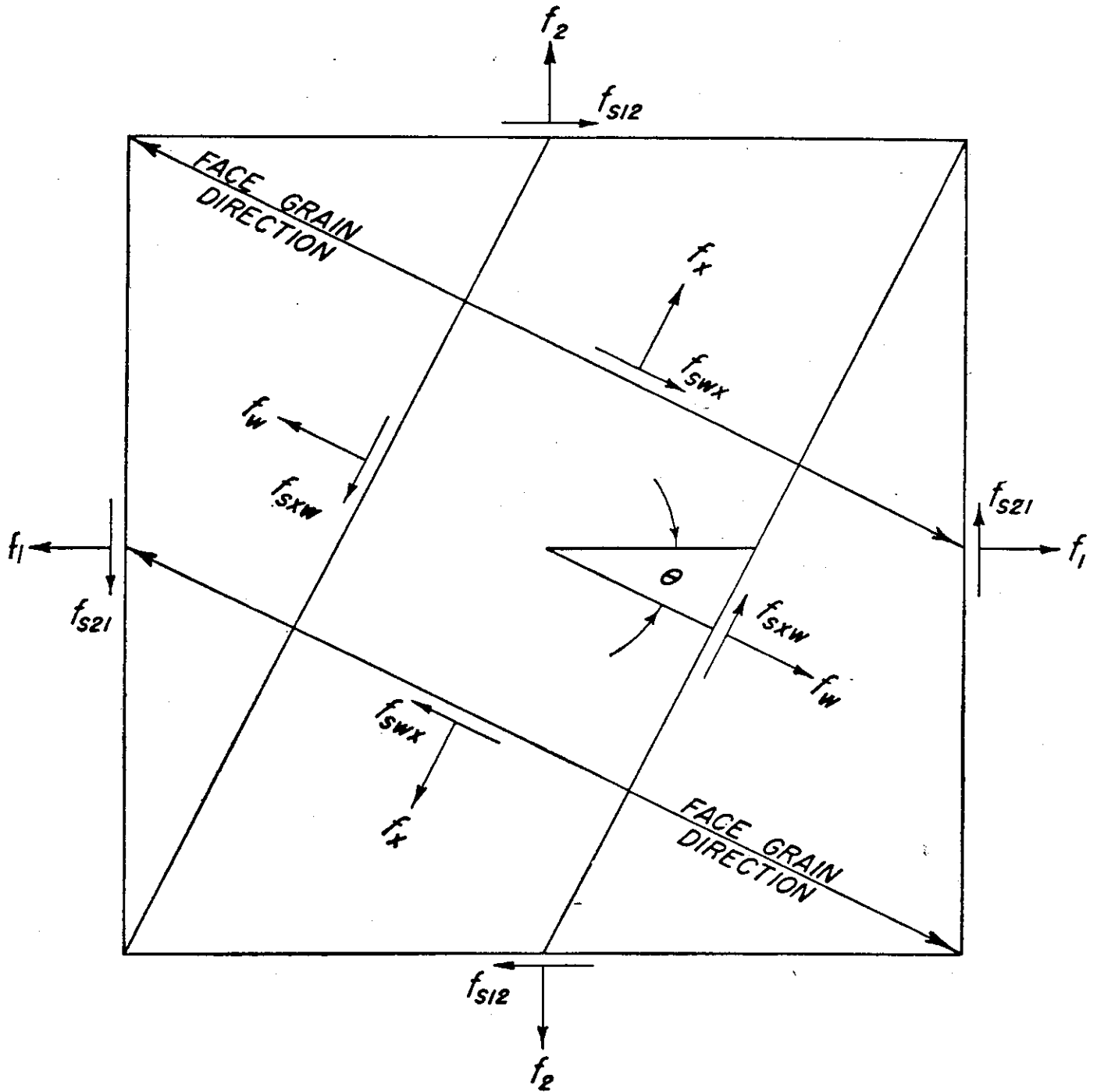


Figure 2-12. General stress distribution in plywood.

If the stresses are known, equations (2:34) completely define the strains. For example, if the plywood is subjected only to a tensile stress f_1 applied at an angle θ to the direction of the face grain, then $f_2=f_{s12}=0$ and equations (2:34) reduce to:

$$\begin{aligned} e_1 &= a_{11} f_1 \\ e_2 &= a_{21} f_1 \\ e_{s12} &= a_{31} f_1 \end{aligned}$$

If the strains are known, equations (2:35) completely define the stresses in a like manner.

The usual transformation equations for stresses and strains apply. The transformation from the arbitrary coordinate system 1, 2 to that of I, II is given by

$$\begin{aligned} f_I &= f_1 \cos^2 \theta + f_2 \sin^2 \theta + 2f_{s12} \sin \theta \cos \theta \\ f_{II} &= f_1 \sin^2 \theta + f_2 \cos^2 \theta - 2f_{s12} \sin \theta \cos \theta \\ f_{sII} &= -f_1 \sin \theta \cos \theta + f_2 \sin \theta \cos \theta + f_{s12} (\cos^2 \theta - \sin^2 \theta) \end{aligned} \quad (2:36)$$

and

$$\begin{aligned} e_I &= e_1 \cos^2 \theta + e_2 \sin^2 \theta + e_{s12} \sin \theta \cos \theta \\ e_{II} &= e_1 \sin^2 \theta + e_2 \cos^2 \theta - e_{s12} \sin \theta \cos \theta \\ e_{sII} &= -2e_1 \sin \theta \cos \theta + 2e_2 \sin \theta \cos \theta + e_{s12} (\cos^2 \theta - \sin^2 \theta) \end{aligned} \quad (2:37)$$

where θ is measured positively from the 01 axis to the 0I axis, as shown in figure 2-13.

Very often the linear strains in three arbitrary directions are known rather than those given in equations (2:34) and (2:35). The required strains can be found by use of the first of equations (2:37) for each of the directions shown in figure 2-14.

The following equations result:

$$\begin{aligned} e_{s12} &= \frac{\frac{e_3}{\sin^2 \theta_3} - \frac{e_4}{\sin^2 \theta_4} - e_1 \left[\frac{\cos^2 \theta_3}{\sin^2 \theta_3} - \frac{\cos^2 \theta_4}{\sin^2 \theta_4} \right]}{\frac{\cos \theta_3}{\sin \theta_3} - \frac{\cos \theta_4}{\sin \theta_4}} \\ e_2 &= \frac{\frac{e_3}{\sin \theta_3 \cos \theta_3} - \frac{e_4}{\sin \theta_4 \cos \theta_4} - e_1 \left[\frac{\cos \theta_3}{\sin \theta_3} - \frac{\cos \theta_4}{\sin \theta_4} \right]}{\frac{\sin \theta_3}{\cos \theta_3} - \frac{\sin \theta_4}{\cos \theta_4}} \end{aligned} \quad (2:38)$$

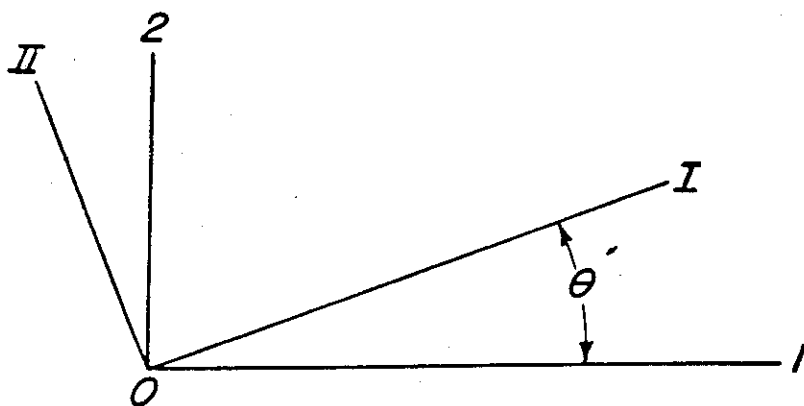


Figure 2-13. Axes for transformation equations.

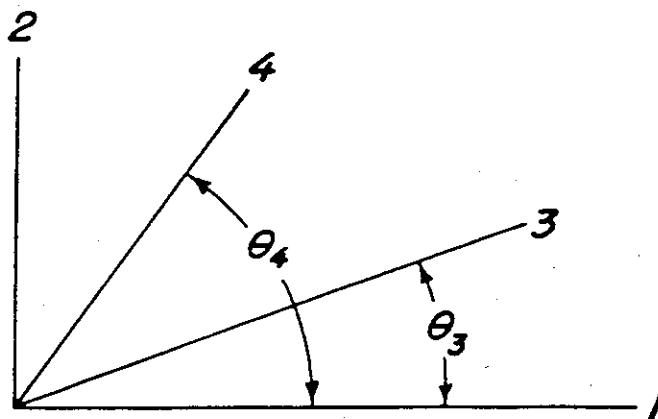


Figure 2-14. Arbitrary directions in which linear strains are known.

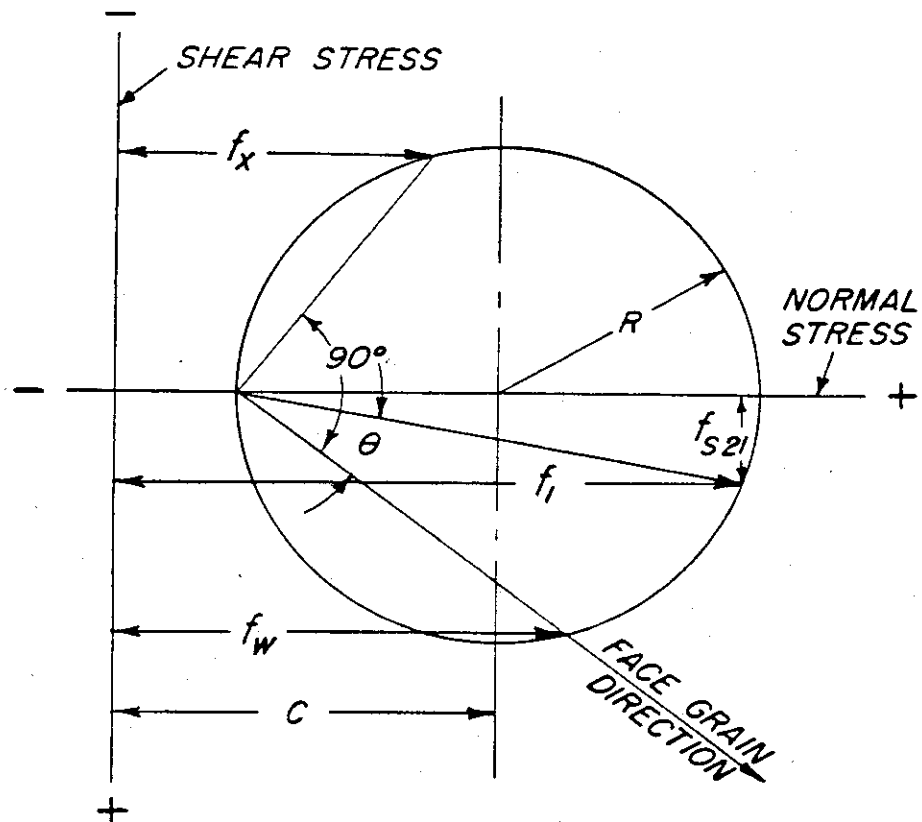


Figure 2-15. Stress circle for stresses shown in figure 2-12.

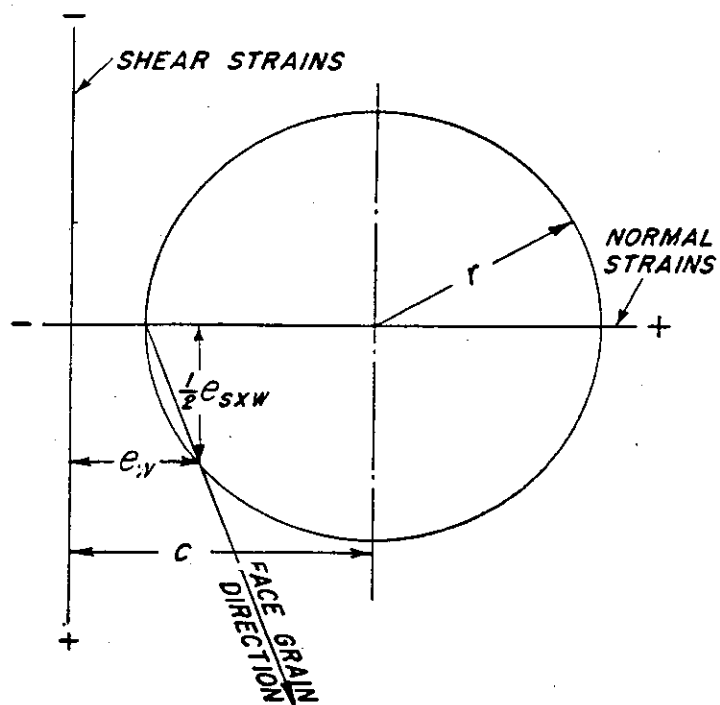


Figure 2-16. Strain circle resulting from equation (2:41).

and if

$$\theta_3 = 45^\circ \text{ and } \theta_4 = 90^\circ,$$

$$e_{s12} = 2e_3 - e_4 - e_1, \quad e_2 = e_4$$

2.560. *Mohr's stress-and-strain circles.* Mohr's stress-and-strain circles are a means of showing, graphically, the relation of stress or strain in one direction to the stress or strain in any other direction and are an aid in the visualization and evaluation of these relations. Reference 2-65 treats extensively the general problem of the use of Mohr's circles in connection with wood and plywood. Only a limited general treatment is presented herein.

2.5600. *Obtaining strains from given stresses.* Assume a stress distribution in a piece of plywood as shown upon the outer square in figure 2-12 (direction of arrows indicates positive direction). The stress circle can be drawn by use of the following equations as shown in figure 2-15, and the stresses parallel and perpendicular to the face grain direction can be determined.

$$C = \frac{1}{2}(f_1 + f_2) \quad (2:39)$$

$$R = \sqrt{(f_1 - C)^2 + (f_{s21})^2} \quad (2:40)$$

The strains parallel and perpendicular to the face grain direction can be found by use of the equations

The stresses parallel and perpendicular to the face grain direction can be obtained from the following equations:

$$f_w = E_w \frac{e_w + e_z \mu_{xz}}{1 - (\mu_{xz} \mu_{zw})} \quad f_z = E_z \frac{e_z + e_w \mu_{wz}}{1 - (\mu_{wz} \mu_{zw})} \quad f_{szw} = G_{wz} e_{szw} \quad (2:45)$$

The stress circle can then be drawn, by the use of the following equations, as shown in figure 2-19, and the stress at any angle to the face grain direction may be found:

$$C = \frac{1}{2}(f_w + f_z) \quad (2:46)$$

$$R = \sqrt{(f_w - C)^2 + (f_{szw})^2} \quad (2:47)$$

2.5602. *Experimental stress-strain data.* Figures 2-20, 2-21, and 2-22 present stress-strain curves of five-ply yellow-poplar plywood subjected to tension, compression, and shear, respectively, at var-

$$e_w = \frac{f_w}{E_w} - \frac{f_z}{E_z} \mu_{zw} \quad e_z = \frac{f_z}{E_z} - \frac{f_w}{E_w} \mu_{wz} \quad e_{szw} = \frac{f_{szw}}{G_{wz}} \quad (2:41)$$

where μ_{wz} and μ_{zw} are given by equations (2:30) and (2:31), respectively.

The strain circle can then be drawn, by the following equations, as shown in figure 2-16 and the strains in any direction can be determined.

$$c = \frac{1}{2}(e_w + e_z) \quad (2:42)$$

$$r = \sqrt{(e_w - c)^2 + \left(\frac{e_{szw}}{2}\right)^2} \quad (2:43)$$

2.5601. *Obtaining stresses from given strains.* The foregoing process can be reversed if strains are given and stresses required. For this purpose strains are usually measured in the three directions shown in figure 2-17. The strain circle can be drawn, by use of the following equations, as shown in figure 2-18, and the strains parallel and perpendicular to the face grain can be found.

$$c = \frac{1}{2}(e_1 + e_2)$$

$$r = \sqrt{\frac{1}{2}(e_1 - e_3)^2 + \frac{1}{2}(e_2 - e_3)^2} \quad (2:44)$$

ious angles to the face grain. These figures are reproduced from reference 2-67.

2.57. STRESS CONCENTRATIONS. (See sections 2-16-2.1611)

2.6. Plywood Structural Elements

The following formulas for strength of plywood elements are applicable only when elastic instability (buckling) is not involved, except in the case of column formulas. For cases involving buckling, see section 2.71.

2.60. ELEMENTS $\theta = (0^\circ \text{ or } 90^\circ)$.

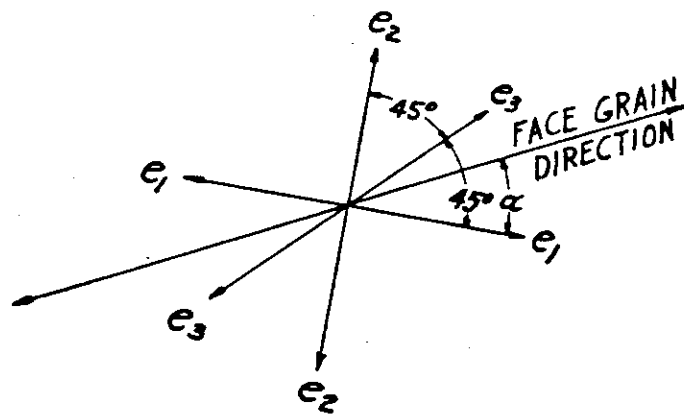


Figure 2-17. Directions of measured strains.

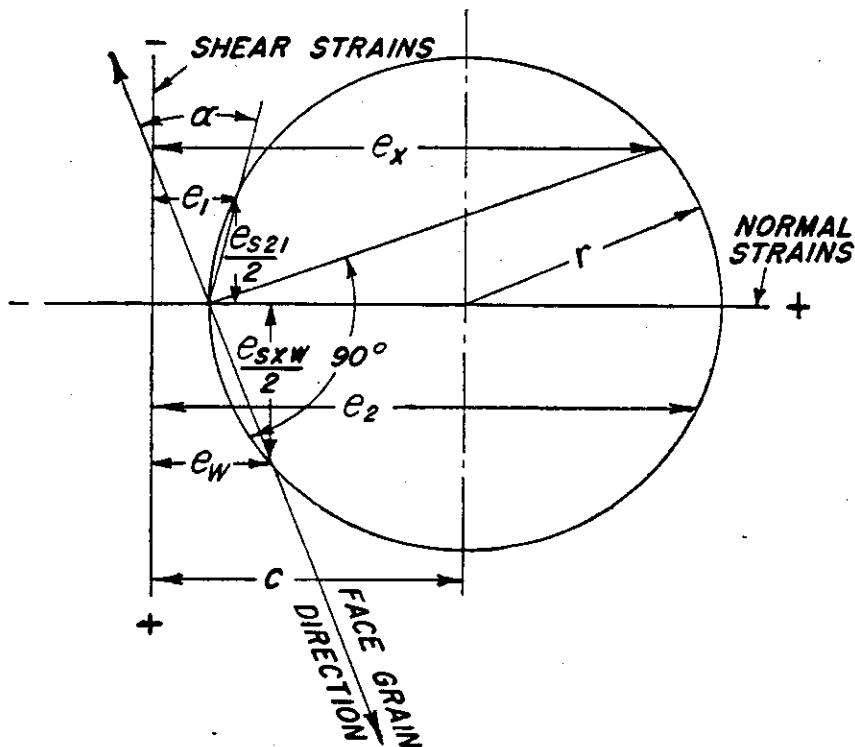


Figure 2-18. Strain circle for measured strains.

2.600. *Elements in compression* ($\theta=0^\circ$ or 90°). When a plywood prism is subjected to a direct compression load, the relation between the internal stress (f_{cL}) in any longitudinal ply and the average P/A stress is given by the following equations: (Ref. 2-43)

Face grain parallel to applied load

$$P/A = f_{cw} = \frac{E_w}{E_{Lc}} f_{cL} \quad (2:48)$$

Face grain perpendicular to applied load

$$P/A = f_{cx} = \frac{E_x}{E_{Lc}} f_{cL} \quad (2:49)$$

The allowable stresses at the proportional limit F_{cpw} and F_{cpz} or the allowable ultimate stresses F_{cuw} and F_{cuz} are obtained from these equations, respectively, when the stress at the proportional limit F_{cp} or the ultimate crushing stress F_{cu} from tables 2-6 or 2-7, whichever is required, is substituted for f_{cL} . When more than one species is used in the longitudinal plies, the species having the lowest ratio of F_{cp}/E_L and F_{cu}/E_L must be used in determining the correct allowables. For certain species and plywood constructions, the compression allowables for the 15 percent moisture content condition may be obtained from table 2-13.

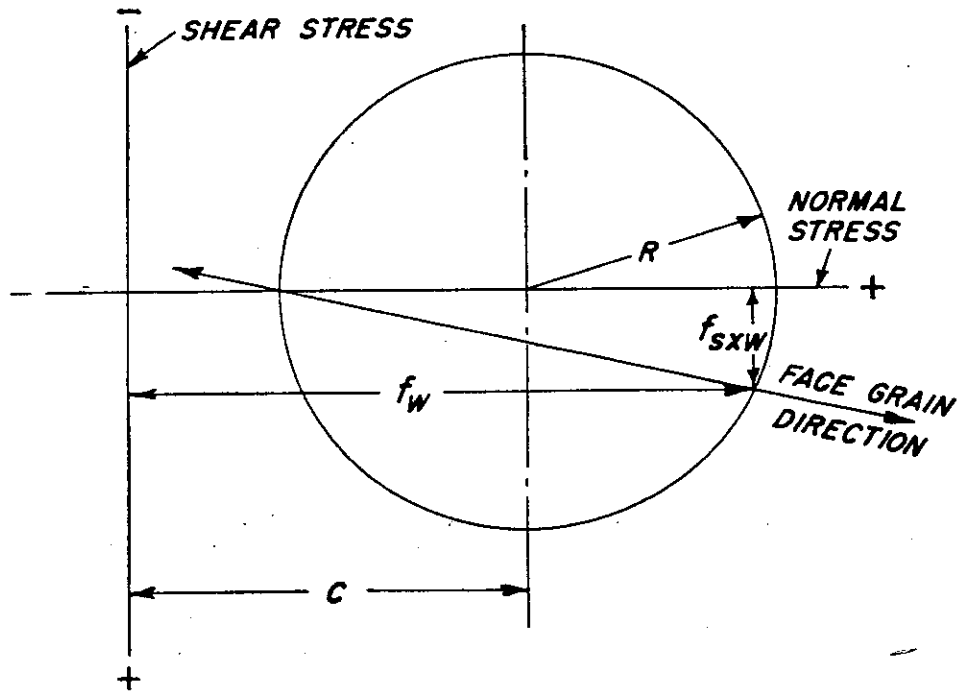


Figure 2-19. Stress circle resulting from equations (2:45)

2.601. *Elements in tension* ($\theta=0^\circ$ or 90°). The allowable ultimate tensile stress for a plywood strip (designated as F_{tuw} when the face grain direction is parallel to the applied load, and F_{tuz} when the face grain is perpendicular to the applied load) is equal to the sum of the strengths of the longitudinal plies divided by the total area of the cross section. The strength of any longitudinal ply is equal to its area multiplied by the modulus of rupture for the species of that ply as given in column 8 of tables 2-6 and 2-7. For certain species and plywood constructions, the tension allowables may be obtained from table 2-13.

2.602. *Elements in shear* ($\theta=0^\circ$ or $\theta=90^\circ$). The allowable ultimate stress of plywood elements subjected to shear is given by the empirical formula: (Ref. 2-88)

$$F_{sz} = 55 \frac{n-1}{t} + \frac{9}{16t} \sum_{i=1}^n F_{sui} t_i \quad (2:50)$$

in which the factor $\frac{n-1}{t}$ shall not be assigned values greater than 35 and in which t_i is the thickness of the i^{th} ply and F_{sui} is the shear strength of wood of the i^{th} ply obtained from tables 2-6 or 2-7. For certain species and plywood constructions the shear allowables for the 15 percent moisture content condition may be obtained from table 2-13.

2.61. ELEMENTS ($\theta=\text{ANY ANGLE}$).

2.610. *Elements in compression* ($\theta=\text{any angle}$). Based upon the results of compression tests of a few species and constructions of plywood, the ultimate compressive stress of narrow elements may be given by:

$$F_{cu\theta} = \frac{1}{\sqrt{\left[\frac{\cos^2\theta}{F_{cuw}}\right]^2 + \left[\frac{\sin^2\theta}{F_{cuz}}\right]^2 + \left[\frac{\sin\theta\cos\theta}{F_{sz}}\right]^2}} \quad (2:51)$$

and the ultimate compressive stress of wide elements, by:

$$F_{cu\theta} = \sqrt{F_{cuw}^2 \cos^4\theta + F_{cuz}^2 \sin^4\theta + 4F_{sz}^2 \sin^2\theta \cos^2\theta} \quad (2:52)$$

where

θ =angle between the face grain and the direction of the applied load.

F_{cuw} =ultimate compressive strength of the plywood parallel to the face grain; from formula (2:48).

F_{cuz} =ultimate compressive strength of the plywood perpendicular to the face grain; from formula (2:49).

F_{sz} =ultimate shear strength of the plywood when the face grain direction is parallel and perpendicular to the shear stresses, from section 2.602.

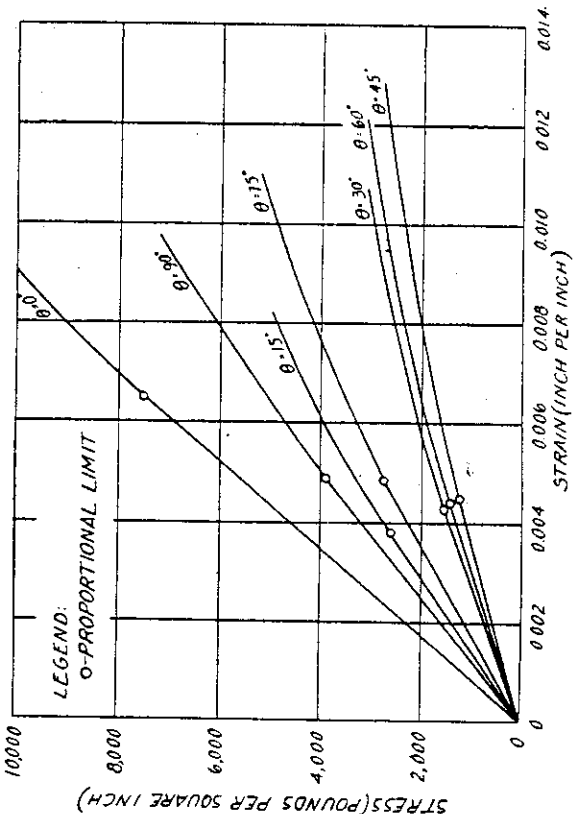
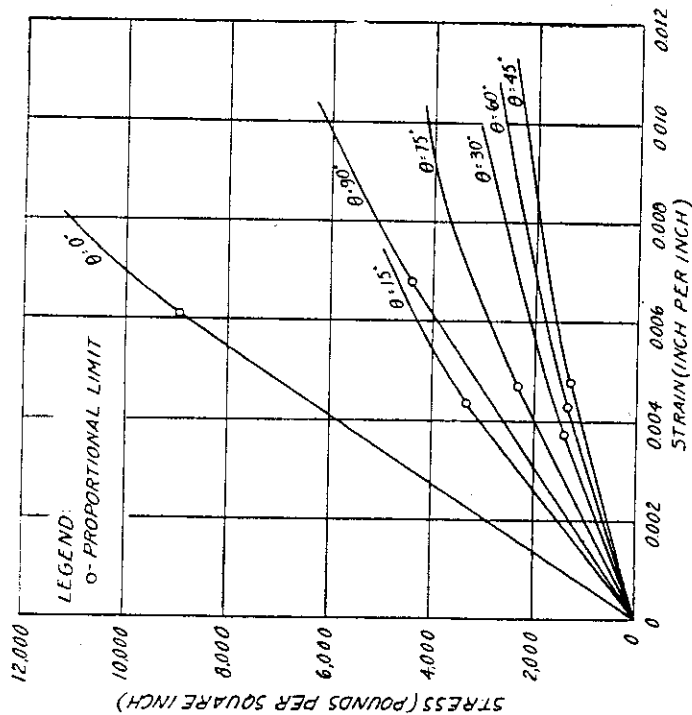


Figure 2-20. Tension tests on yellow-poplar plywood at various angles to the face grain. Plywood was made of five $\frac{1}{8}$ -inch plies. Each curve represents values from 16 tests on specimens 0.8 inches in width.

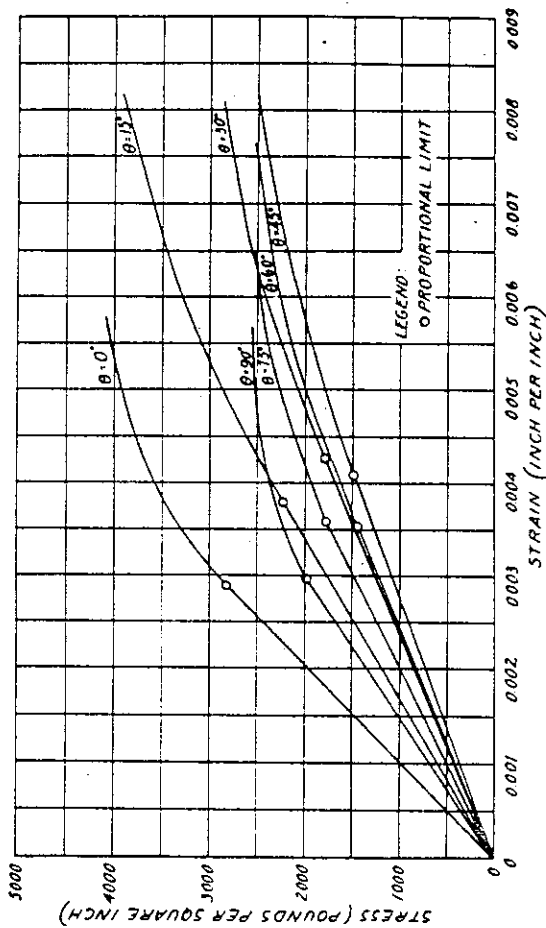
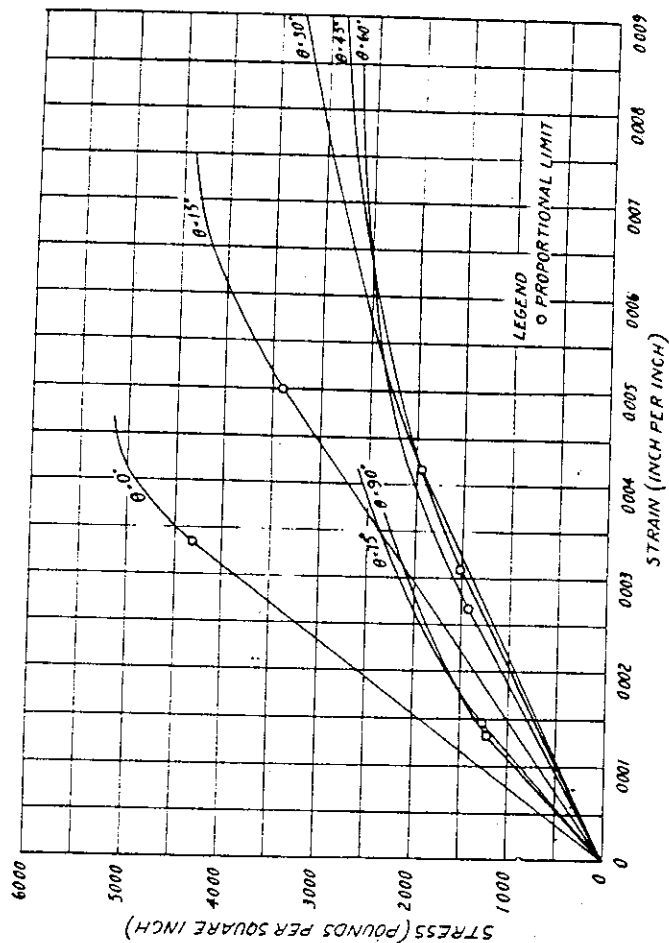


Figure 2-21. Stress-strain curves from compression tests on yellow-poplar plywood at various angles to the face grain. Plywood was made of five $\frac{1}{8}$ -inch plies. Each curve represents values from four tests on specimens 6 inches in width.

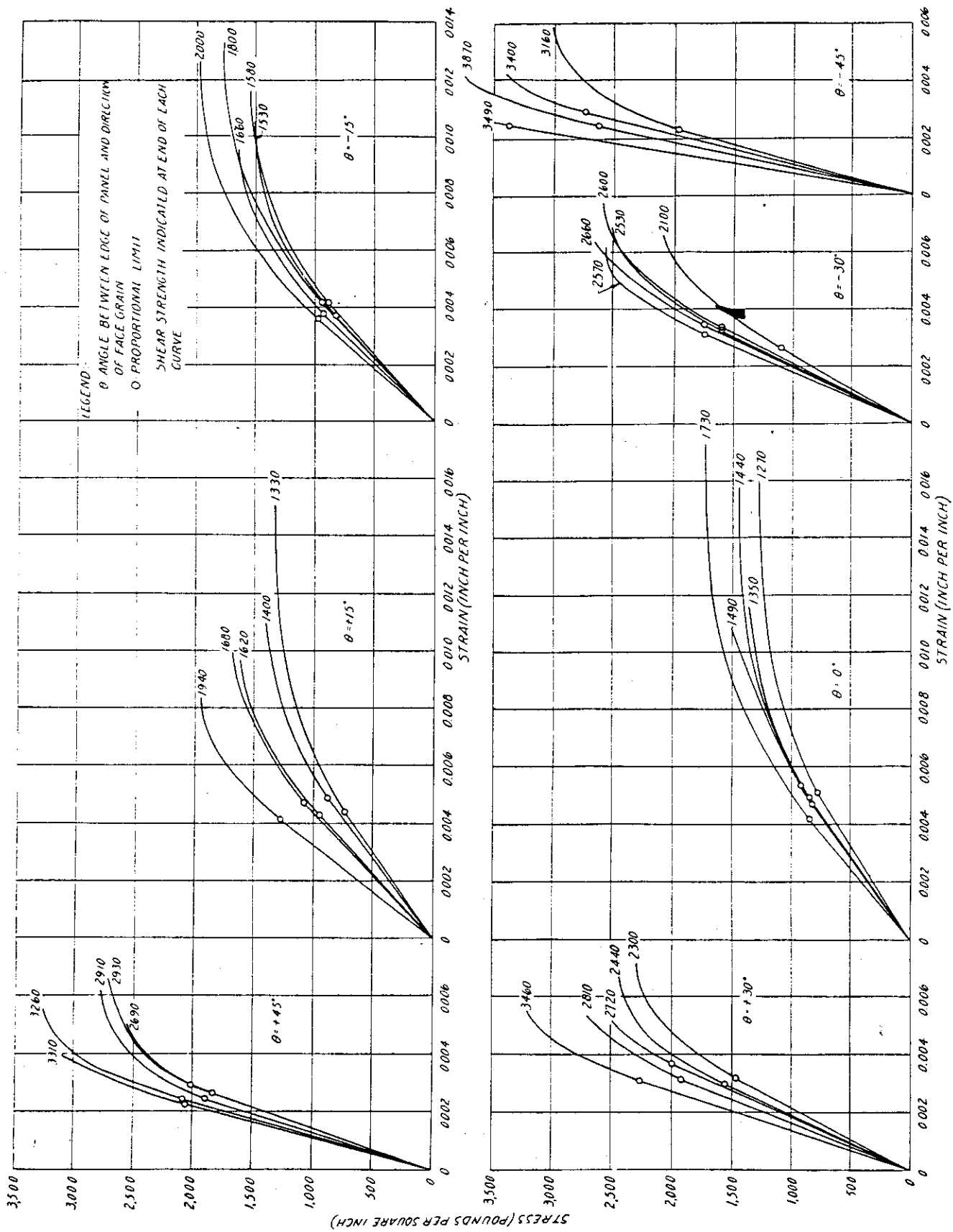


Figure 2-22. Panel shear tests on yellow-poplar plywood loaded at various angles to the face grain. Strains were measured on the principal axis of compression. Construction was five plies of $\frac{1}{16}$ -inch veneer.

2.611. *Elements in tension* (θ =any angle). The ultimate tensile strength of narrow elements is given by the formula:

$$F_{tu\theta} = \frac{1}{\sqrt{\left[\frac{\cos^2 \theta}{F_{tuw}}\right]^2 + \left[\frac{\sin^2 \theta}{F_{tuz}}\right]^2 + \left[\frac{\sin \theta \cos \theta}{F_{suw}}\right]^2}} \quad (2:53)$$

and the tensile strength of wide elements, by

$$F_{tu\theta} = \sqrt{F_{tuw}^2 \cos^4 \theta + F_{tuz}^2 \sin^4 \theta + 4 F_{suw} \sin^2 \theta \cos^2 \theta} \quad (2:54)$$

where

F_{tuw} and F_{tuz} =ultimate tensile strength of plywood parallel and perpendicular to the face grain direction, respectively, from section 2.601.

2.612. *Elements in shear* (θ =any angle). The ultimate shear strength of plywood in this case is given by equations (2:55) and (2:56). When shear tends to place the face grain in tension, equation (2:55) should be used. When shear tends to place the face grain in compression, equation (2:56) should be used.

When face grain is in tension

$$F_{s\theta t} = \frac{1}{\sqrt{\left(\frac{1}{F_{tuw}^2} + \frac{1}{F_{cuw}^2}\right) \sin^2 2\theta + \frac{\cos^2 2\theta}{F_{suw}^2}}} \quad (2:55)$$

When face grain is in compression

$$F_{s\theta c} = \frac{1}{\sqrt{\left(\frac{1}{F_{cuw}^2} + \frac{1}{F_{tuz}^2}\right) \sin^2 2\theta + \frac{\cos^2 2\theta}{F_{suw}^2}}} \quad (2:56)$$

For the special case of the face grain at 45° to the side of the panel, equations (2:55) and (2:56) reduce to

$$F_{s45t} = \frac{F_{tuw}}{\sqrt{1 + \left(\frac{F_{tuw}}{F_{cuw}}\right)^2}} \quad (2:57)$$

$$F_{s45c} = \frac{F_{tuz}}{\sqrt{1 + \left(\frac{F_{tuz}}{F_{cuw}}\right)^2}} \quad (2:58)$$

2.613. *Elements in combined compression (or tension) and shear* (θ =any angle). The condition

for failure of plywood elements subjected to combined stresses in the plane of the plywood is given by the following equation. Formulas (2:53) to (2:58) are special cases of this general equation.

$$\left(\frac{f_w}{F_w}\right)^2 + \left(\frac{f_z}{F_z}\right)^2 + \left(\frac{f_{suw}}{F_{suw}}\right)^2 = 1 \quad (2:59)$$

where

f_w/F_w =ratio of the internal tension or compression stress, parallel to the face grain, to the allowable tension or compression stress in the same direction.

f_z/F_z =ratio of the internal tension or compression stress, perpendicular to the face grain, to the allowable tension or compression stress in the same direction.

f_{suw}/F_{suw} =ratio of the internal shear stress, parallel and perpendicular to the face grain, to the allowable shear stress in the same direction.

In the use of equation (2:59), it is necessary first to resolve the internal stresses into directions that are parallel and perpendicular to the face grain direction by use of the following transformation equations:

$$f_w = f_1 \cos^2 \theta + f_2 \sin^2 \theta - 2f_{s12} \sin \theta \cos \theta$$

$$f_z = f_1 \sin^2 \theta + f_2 \cos^2 \theta + 2f_{s12} \sin \theta \cos \theta$$

$$f_{suw} = (f_1 - f_2) \sin \theta \cos \theta + f_{s12} (\cos^2 \theta - \sin^2 \theta) \quad (2:60)$$

in which the symbols have the meanings indicated in figure 2-12 and θ is measured positively from the direction of the face grain to the direction of axis 1.

In order to clarify the use which can be made of the combined loading equation (2:59), the complete derivation of equation (2:53) is given. It is desired to find the allowable tensile stress of a plywood element which is loaded as shown in figure 2-23.

Thus in equation (2:60) $f_w = f_{tw}$, $f_z = f_{tz}$, $f_1 = f_t$ and $f_2 = f_{s12} = 0$; and the equations reduce to:

$$f_{tw} = f_t \cos^2 \theta$$

$$f_{tz} = f_t \sin^2 \theta$$

$$f_{suw} = f_t \sin \theta \cos \theta$$

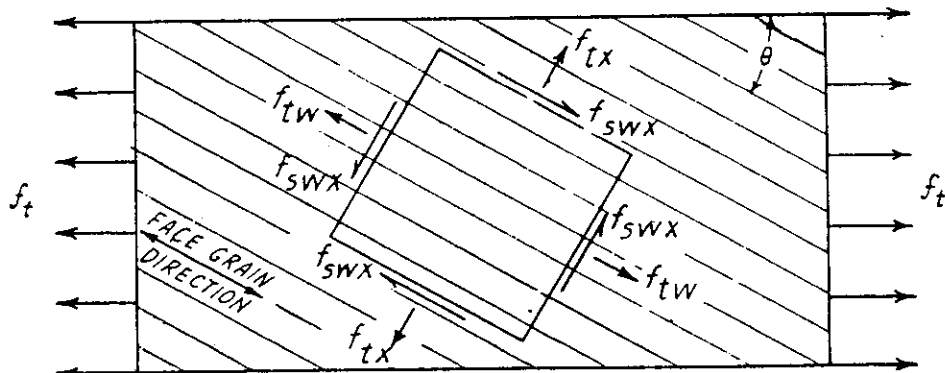


Figure 2-23. Orientation of plywood element for derivation of formula (2:53).

Substituting these terms in the combined loading equation

$$\left[\frac{f_{tw}}{F_{tw}}\right]^2 + \left[\frac{f_{tx}}{F_{tx}}\right]^2 + \left[\frac{f_{swx}}{F_{swx}}\right]^2 = 1$$

the following is obtained:

$$\left[\frac{f_t \cos^2 \theta}{F_{tw}}\right]^2 + \left[\frac{f_t \sin^2 \theta}{F_{tx}}\right]^2 + \left[\frac{f_t \sin \theta \cos \theta}{F_{swx}}\right]^2 = 1$$

Dividing through by f_t and setting its value equal to the allowable tensile stress, $F_{tw}\theta$, gives equation (2:53), or

$$F_{tw}\theta = \frac{1}{\sqrt{\left[\frac{\cos^2 \theta}{F_{tw}}\right]^2 + \left[\frac{\sin^2 \theta}{F_{tx}}\right]^2 + \left[\frac{\sin \theta \cos \theta}{F_{swx}}\right]^2}} \quad (2:53)$$

Equations (2:51), (2:55), and (2:56) may be derived in exactly the same manner.

Equation (2:38) is derived in a similar manner in which f_2 and f_{s12} are not equated to zero but are allowed to assume values which make f_t a maximum. Equation (2:37) also may be derived in this manner.

These equations were taken from reference 2-67.

2.614. *Elements in bending.* The apparent moduli of elasticity (E_{fw} and E_{fx}) of plywood beams in bending are given by the general formulas in section 2.5. When all of the plies are of equal thickness and one species, these general formulas reduce to the following forms:

For rotary-cut veneer,

$$\text{three-ply; } E_{fw} = \frac{E_L}{27} \left(\frac{E_T}{E_L} + 26 \right) \quad (2:61)$$

$$E_{fx} = \frac{E_L}{27} \left(1 + 26 \frac{E_T}{E_L} \right)$$

$$\text{five-ply; } E_{fw} = \frac{E_L}{125} \left(26 \frac{E_T}{E_L} + 99 \right) \quad (2:62)$$

seven-ply;

$$E_{fx} = \frac{E_L}{125} \left(26 + 99 \frac{E_T}{E_L} \right)$$

$$E_{fw} = \frac{E_L}{343} \left(99 \frac{E_T}{E_L} + 244 \right)$$

$$E_{fx} = \frac{E_L}{343} \left(99 + 244 \frac{E_T}{E_L} \right) \quad (2:63)$$

nine-ply;

$$E_{fw} = \frac{E_L}{729} \left(244 \frac{E_T}{E_L} + 485 \right)$$

$$E_{fx} = \frac{E_L}{729} \left(244 + 485 \frac{E_T}{E_L} \right) \quad (2:64)$$

For quarter-sliced veneer, E_T/E_L should be replaced by E_R/E_L (sec. 2.13).

The bending stress in the extreme fiber of the outermost longitudinal ply is given by the following formulas:

Face grain parallel to span

$$f_b = 1.18 \frac{Mc'}{I} \frac{E_L}{E_{fw}} \quad (2:65)$$

Face grain perpendicular to span

$$f_b = 0.91 \frac{Mc'}{I} \frac{E_L}{E'_{fx}} \quad (\text{for 3-ply}) \quad (2:66)$$

$$f_b = 1.11 \frac{Mc'}{I} \frac{E_L}{E'_{fx}} \quad (\text{all other}) \quad (2:67)$$

where

c' = distance from neutral axis to extreme fiber of outermost longitudinal ply.

E'_{fx} = same as E_{fx} except that outermost ply in tension is neglected. E_{fx} may be used in place of E'_{fx} in formula (2:67) with only slight error.

E_L is taken for the species of the outermost longitudinal ply.

The allowable bending stress at proportional limit (F_{bp}) and the modulus of rupture in bending (F_{bu}) are given in tables 2-6 and 2-7. (Ref. 2-25)

2.6140. *Deflections.* The deflection of plywood beams with face grain parallel or perpendicular to the span may be obtained by using E_{fw} or E_{fx} in the ordinary beam formulas. E'_{fx} is used only for determining strengths in bending and not the deflection. For plywood beams with face grain at an angle θ to the direction of the span, the effective modulus of elasticity to be used in the deflection formula is given by the equation:

$$E = \frac{1}{\lambda_f} [E_{fu} \cos^4 \theta + E_{fx} \sin^4 \theta + (2 E_{fu} \mu_{fxu} + 4 \lambda_f G_{fux}) \sin^2 \theta \cos^2 \theta] \quad (2:68)$$

in which $\lambda_f = 1 - \mu_{fxu} \mu_{fxv}$
when

- (1) The loading is constant across the width of the beam at any point in its span.
- (2) The beam width is sufficient to cause the deflection to be constant across the beam at any point in the span.
- (3) The beam is held so that it cannot leave the supports.

There are no methods available by which the strength of plywood beams may be calculated

when the grain direction of the face plies is other than parallel or perpendicular to the span.

2.615. *Elements as columns.* The allowable stresses for plywood columns are given by the following formulas:

Long columns

$$F_c = \frac{0.85 \pi^2 E_{fw}}{(L'/\rho)^2} \quad (\text{face grain parallel to length}) \quad (2:69)$$

$$F_c = \frac{0.85 \pi^2 E_{fx}}{(L'/\rho)^2} \quad (\text{face grain perpendicular to length}) \quad (2:70)$$

$$(L'/\rho)_{cr} = 3.55 \sqrt{\frac{E_{fw}}{E_{cuw}}}$$

$$\text{or } 3.55 \sqrt{\frac{E_{fx}}{E_{cux}}} \text{ respectively}$$

Short columns

$$F_c = F_{cu} \left[1 - \frac{1}{3} \left(\frac{L'}{K\rho} \right)^4 \right] \quad (2:71)$$

where

$$\begin{aligned} K &= (L'/\rho)_{cr} \\ F_{cu} &= F_{cuw} \text{ when face grain is parallel to length} \\ &= F_{cux} \text{ when face grain is perpendicular to length} \end{aligned}$$

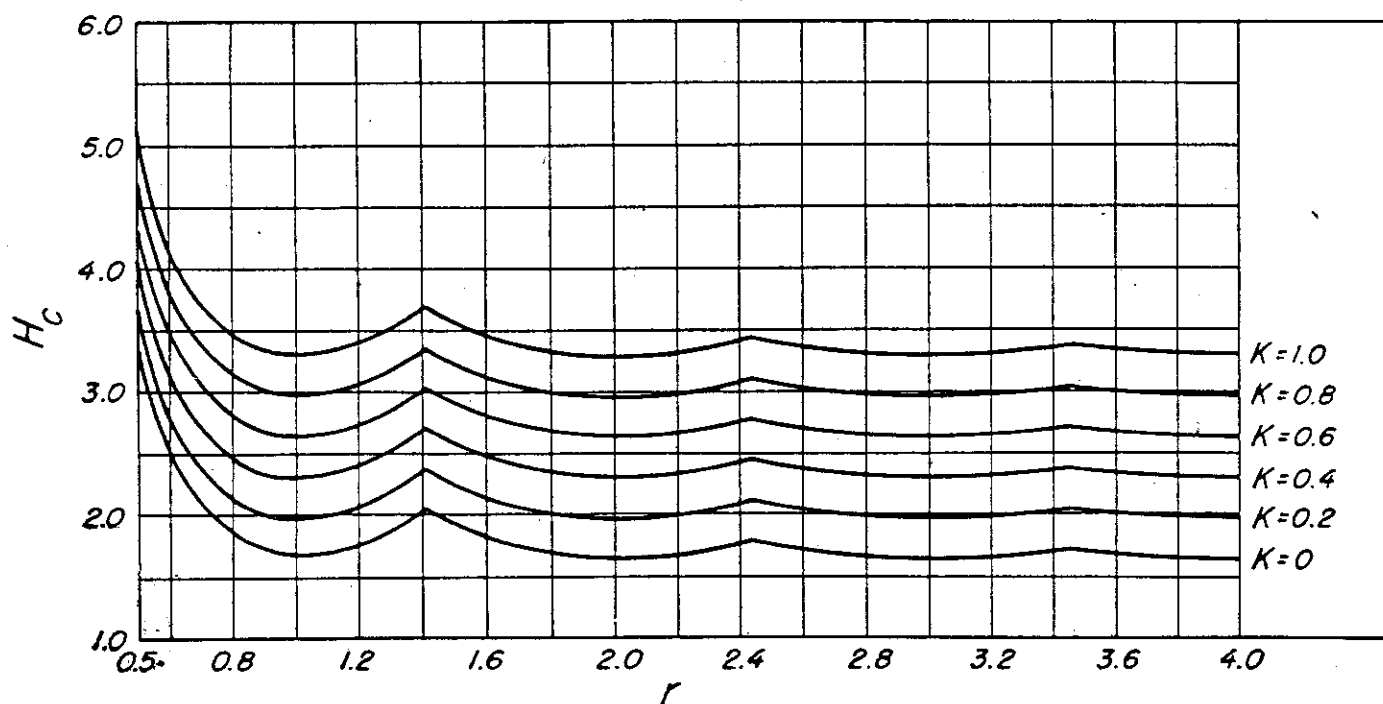


Figure 2-25. Plot of equation (2:74).

2.7. Flat Rectangular Plywood Panels

2.71. BUCKLING CRITERIA.

2.711. *General.* When buckling occurs in plywood panels at loads less than the required design loads, the resulting redistribution of stresses must be considered in the analysis of the structure. The buckling criteria in this section are based on mathematical analyses and are confirmed by experiments for stresses below the proportional limit. Visible buckling may occur at lower stresses than those indicated by these criteria, due to the imperfections and eccentric loadings which usually exist in structures. Experiments have indicated, however, that the redistribution of stresses due to buckling corresponds more closely to the degree of buckling indicated by these theoretical criteria than it does to visible buckling. These criteria can, therefore, be used in various parameters for plotting test results or design allowables against the degree of buckling, and to compute the degree of buckling in a structure. This is done in sections 2.72 and 2.760.

Since the mathematical analyses are based on the assumption of elastic behavior, these criteria cannot be directly applied when the stresses are above proportional limit. The behavior at such stresses has been investigated experimentally for some cases, as described in sections 2.72 and 2.760. Because of the low modulus of elasticity of wood across the grain it is difficult to approach clamped-edge conditions.

2.712. *Compression* ($\beta=0^\circ$ or 90°). The critical buckling stress of flat rectangular plywood panels subjected to uniform compressive stress is given by the following formula (ref. 2-55).

$$F_{cer} = H_c \frac{\sqrt{E_{fx} E_{fy}}}{\lambda_f} \left(\frac{t}{a}\right)^2 \tag{2.72}$$

in which H_c depends upon the edge conditions of the panel and other considerations in the following cases.

Case I. All edges simply supported.

$$H_c = \frac{\pi^2}{12} \left[\left(\frac{r}{m}\right)^2 + \left(\frac{m}{r}\right)^2 + 2k \right] \tag{2.73}$$

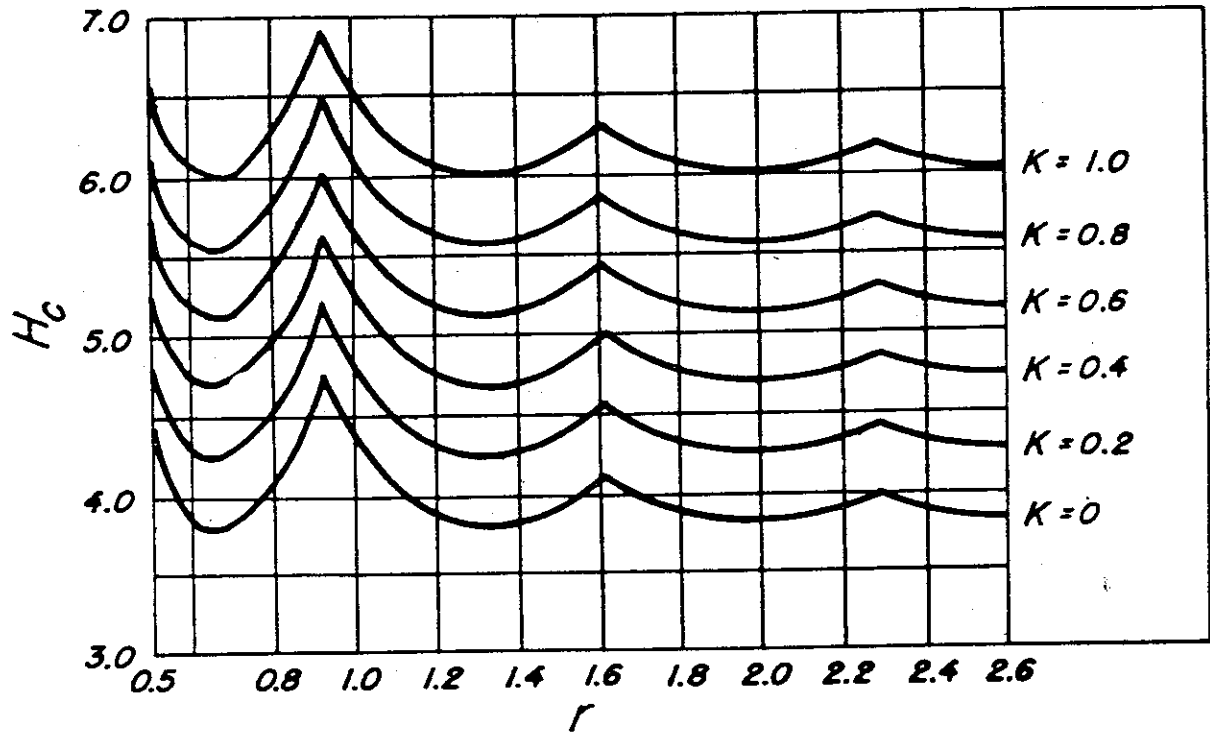


Figure 2-24. Plot of equation (2.73)

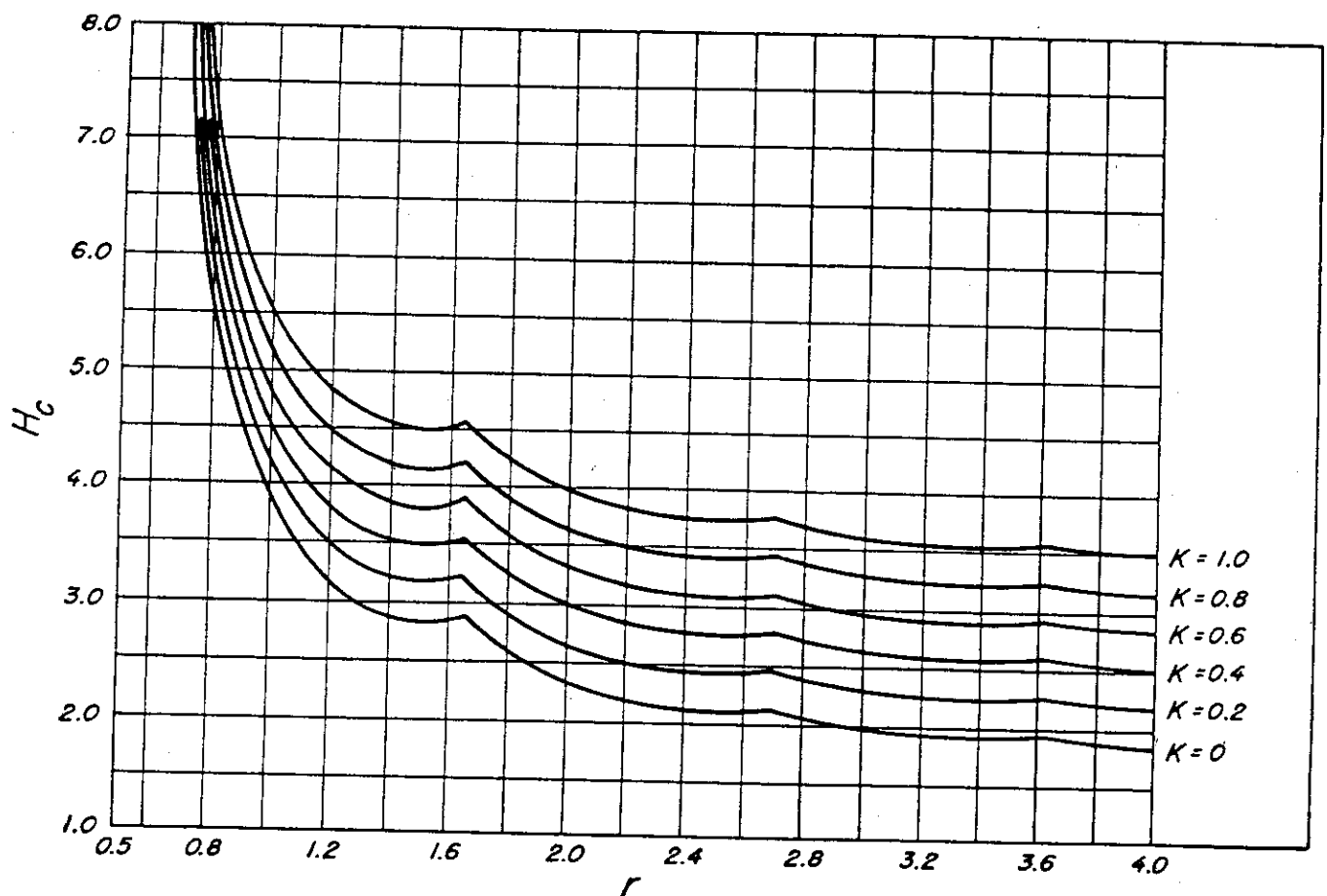


Figure 2-26. Plot of equation (2:75).

in which

$$k = \frac{E_{fzu} \mu_{fzu} + 2\lambda_f G_{fuz}}{\sqrt{E_{fzu} E_{fz}}}$$

$$r = \frac{b}{a} \left(\frac{E_1}{E_2} \right)^{1/4}$$

m = the number of half-wave lengths in the shape of the buckled panel defined by the inequality,

$$\sqrt{m(m-1)} < r < \sqrt{m(m+1)}$$

Case II. Loaded edges simply supported. Remaining edges clamped.

$$H_c = \frac{4\pi^2}{9} \left[\left(\frac{r}{m} \right)^2 + \frac{3}{16} \left(\frac{m}{r} \right)^2 + \frac{k}{2} \right] \quad (2:74)$$

$$\frac{1}{2} \sqrt{m(m-1)} \sqrt{3} < r < \frac{1}{2} \sqrt{m(m+1)} \sqrt{3}$$

Case III. Two loaded edges clamped. Remaining edges simply supported.

For one half-wave

$$H_c = \frac{\pi^2}{48} \left(3r^2 + \frac{16}{r^2} + 8k \right)$$

For two half-waves

$$H_c = \frac{\pi^2}{60} \left(r^2 + \frac{41}{r^2} + 10k \right)$$

For three half-waves (2:75)

$$H_c = \frac{\pi^2}{120} \left(r^2 + \frac{136}{r^2} + 20k \right)$$

For four half-waves

$$H_c = \frac{\pi^2}{204} \left(r^2 + \frac{353}{r^2} + 34k \right)$$

The value of H_c that is least for the particular panel involved, should be used.

Case IV. All edges clamped.

For one half-wave

$$H_c = \frac{\pi^2}{9} \left(3r^2 + \frac{3}{r^2} + 2k \right)$$

For two half-waves

$$H_c = \frac{\pi^2}{180} \left(16r^2 + \frac{123}{r^2} + 40k \right)$$

For three half-waves

$$H_c = \frac{\pi^2}{45} \left(2r^2 + \frac{51}{r^2} + 10k \right)$$

For four half-waves

$$H_c = \frac{\pi^2}{612} \left(16r^2 + \frac{1059}{r^2} + 136k \right) \quad (2:76)$$

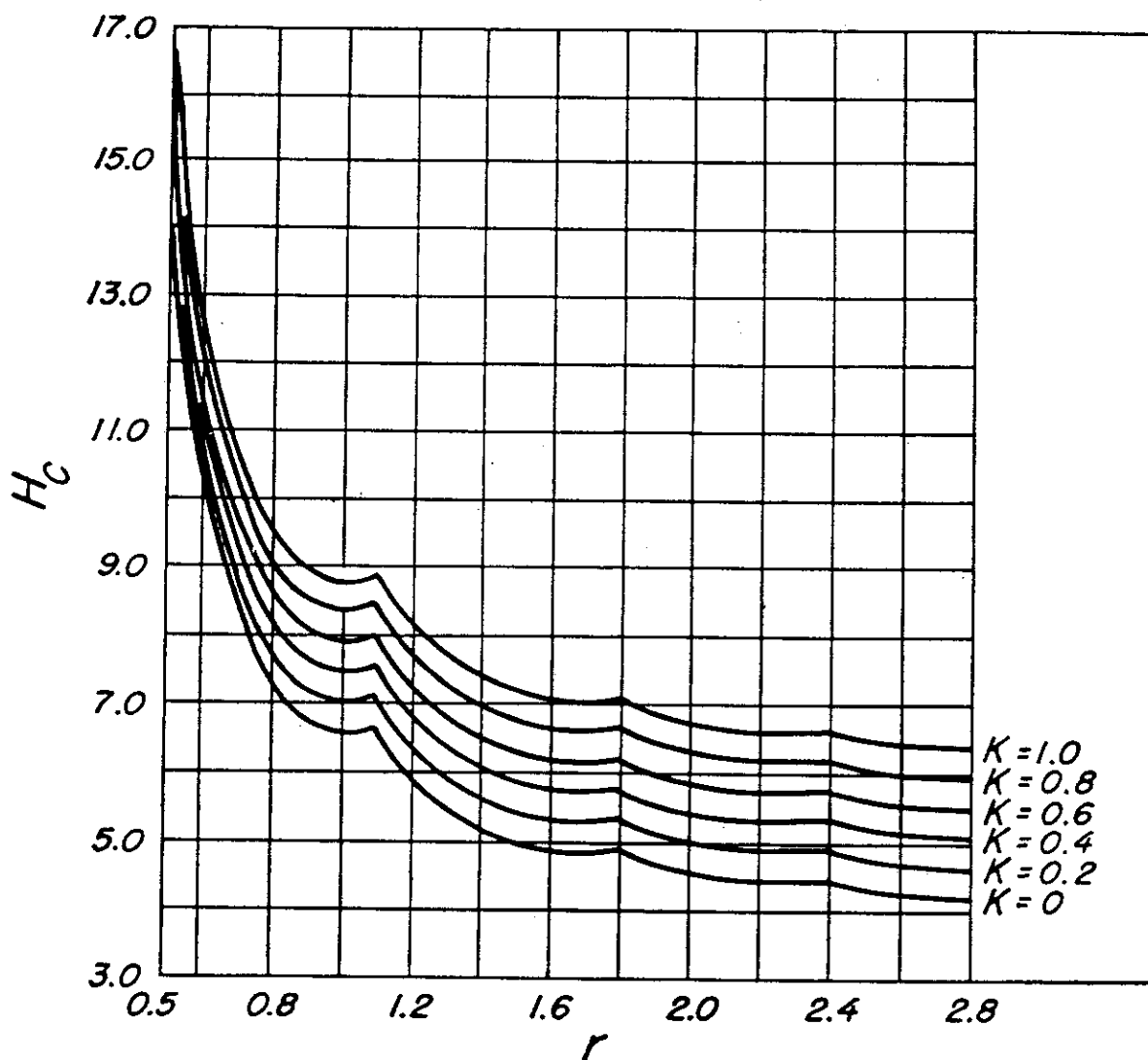


Figure 2-27. Plot of equation (2:76).

The value of H_c that is least for the particular panel involved, should be used.

2.713. *Shear* ($\beta=0^\circ$ or 90°). The critical buckling stress of flat rectangular plywood panels subjected to uniform shear stress is given by the following formula (ref. 2-50).

$$F_{scr} = H_s \frac{(E_1^3 E_2)^{1/4}}{3\lambda_f} \left(\frac{t}{a}\right)^2 \quad (2:77)$$

in which values of H_s are obtained from figure 2-28 for panels having simply supported edges and from figure 2-29 for clamped edges. The symbols a and b are assigned to the dimensions of the panel

in such a way that $\frac{1}{r} = \frac{a}{b} \left(\frac{E_2}{E_1}\right)^{1/4}$ is less than or equal to unity.

2.714. *Combined compression and shear* ($\beta=0^\circ$ or 90°). The critical buckling stresses of flat rectangular panels subjected to combined compression and shear may be obtained by use of the fol-

lowing interaction formula (ref. 2-50)

$$\left(\frac{f_{scr}}{F_{scr}}\right)^2 + \frac{f_{ccr}}{F_{ccr}} = 1 \quad (2:78)$$

in which the ratio of f_{scr} to f_{ccr} is given by the particular problem to be solved. This equation is accurate only for isotropic plates, but is not greatly in error for plywood if $\beta=0^\circ$ or 90° .

2.715. *Compression, shear, and combined compression and shear* ($\beta=\text{any angle}$). When the direction of the face grain of the plywood makes an angle other than 0° or 90° with the edge of the plywood, accurate methods of calculation are extremely complicated, and, therefore, an approximate method is resorted to. The curves of figures 2-30 to 2-41, inclusive, apply accurately only to Douglas-fir plywood, but their use is extended approximately to other species by the method described. Curves for the cases in which $\beta=0^\circ$ or 90° are included for the sake of completeness.

By this method the critical buckling stress of flat rectangular plywood panels subjected to either uniform compression or uniform shear stresses is given by the following general formulas.

$$F_{scr} = K_c [E_{fw} + E_{fz}] \left(\frac{t}{a} \right)^2 \quad (2:79)$$

$$F_{scr} = K_s [E_{fw} + E_{fz}] \left(\frac{t}{a} \right)^2 \quad (2:80)$$

where

K_c and K_s are factors depending on the type of loading, the dimensions of the panel, the edge-fixity conditions, and Poisson's ratio. K_c and K_s are determined by the following methods.

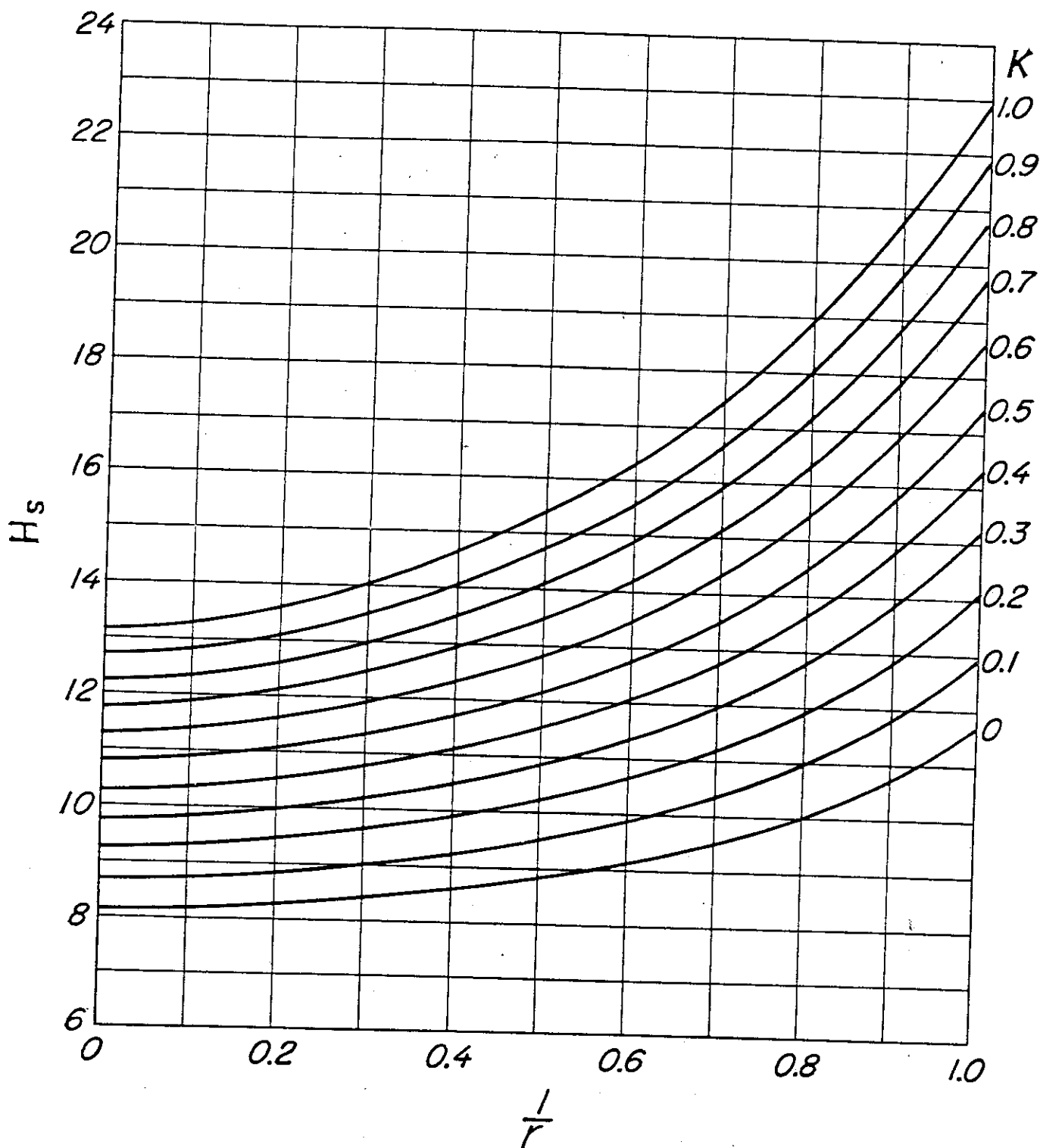


Figure 2-28. Curves for calculating the buckling shear stress in orthotropic rectangular plates, with simply supported edges whose axes of elastic symmetry are parallel to the edges. (Taken from paper by E. Seydel, *Zeitschrift für Flugtechnik und Luftschiffahrt* 24, 78-83, 1933.)

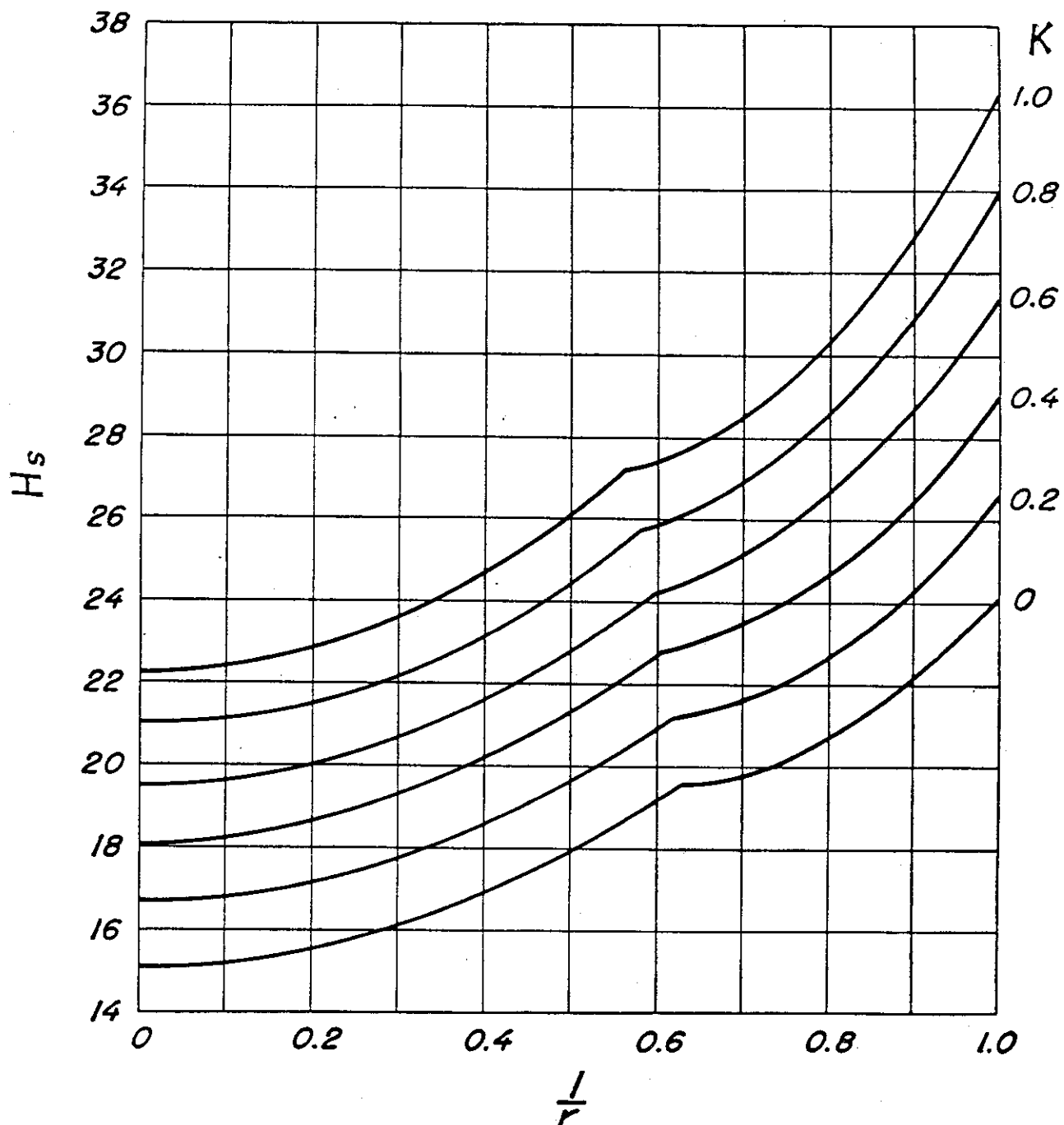


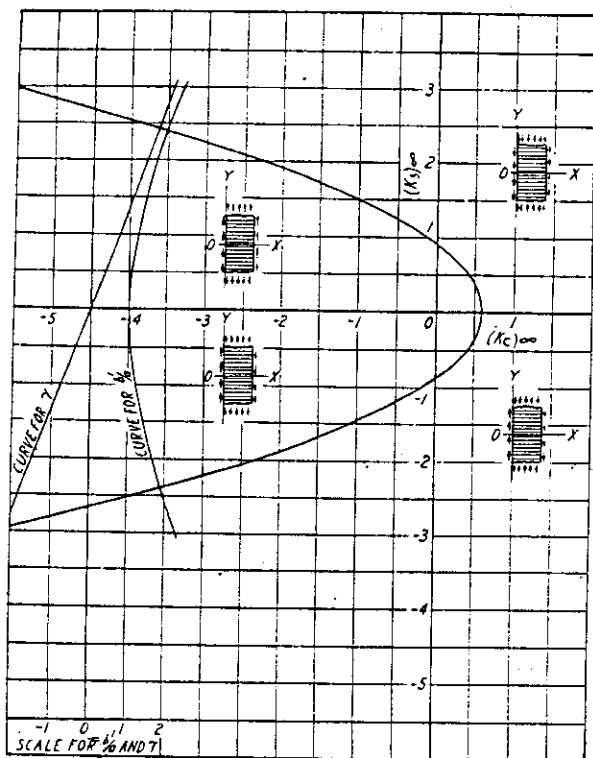
Figure 2-29. Curves for calculating the buckling shear stress in orthotropic rectangular plates, with clamped edges, whose axes of elastic symmetry are parallel to the edges. (Taken from report by R. C. T. Smith "The Buckling of Plywood Plates in Shear," Report SM. 51 of the Council for Scientific and Industrial Research, Division of Aeronautics Commonwealth of Australia.)

Let a be the width of a rectangular panel of infinite length of which a portion of finite length b is being considered.

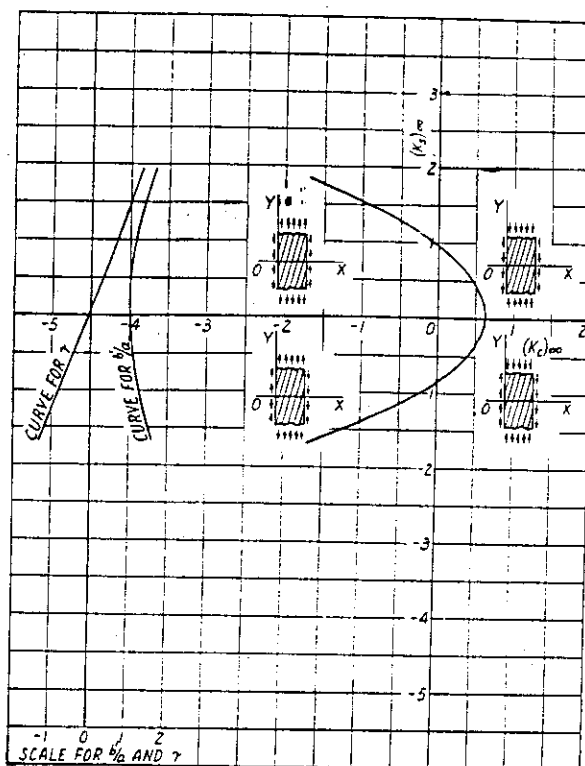
The mathematical treatment of buckling constants presented in this section has been based on the assumption that the compression load is always placed on the edge having dimension a . In a panel loaded only in shear a dimension of either edge may be taken as a , and the panel

shall be considered as a $\beta=0^\circ$ case when the face grain is perpendicular to the edge having dimension a and as a $\beta=90^\circ$ case when the face grain is parallel to the edge having dimension a (fig. 2-42).

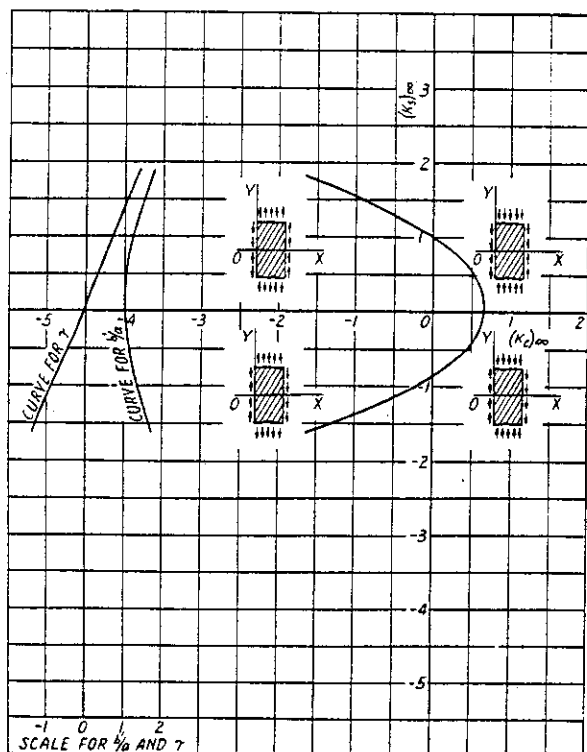
One method of obtaining K_s or K_c is by the use of figures 2-30 to 2-36 as explained in section 2.7151. Approximate values of K_s or K_c suitable for ordinary purposes may be obtained by cor-



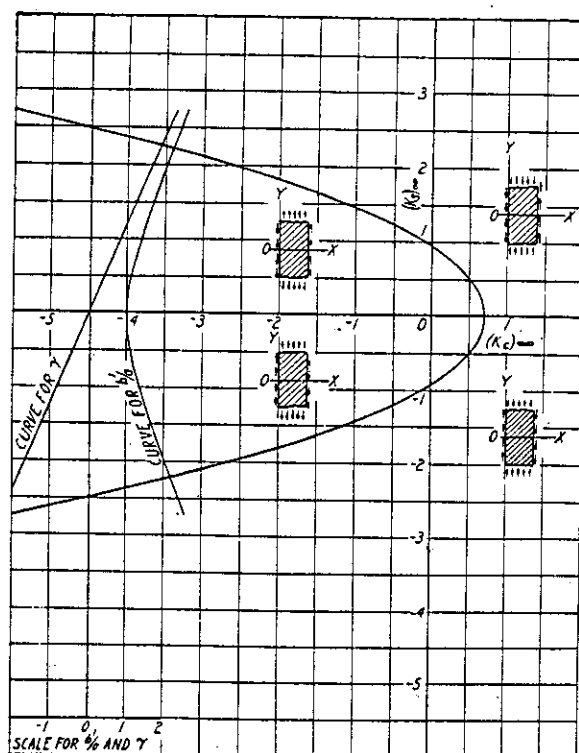
(a)
2-PLY (1:1) $\beta = 0^\circ$ AND $\beta = 90^\circ$



(b)
2-PLY (1:1) $\beta = 15^\circ$

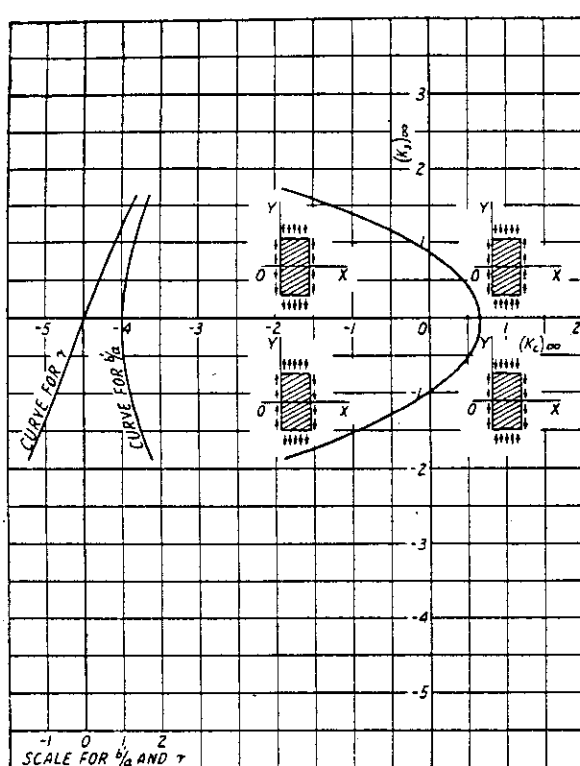


(c)
2-PLY (1:1) $\beta = 30^\circ$

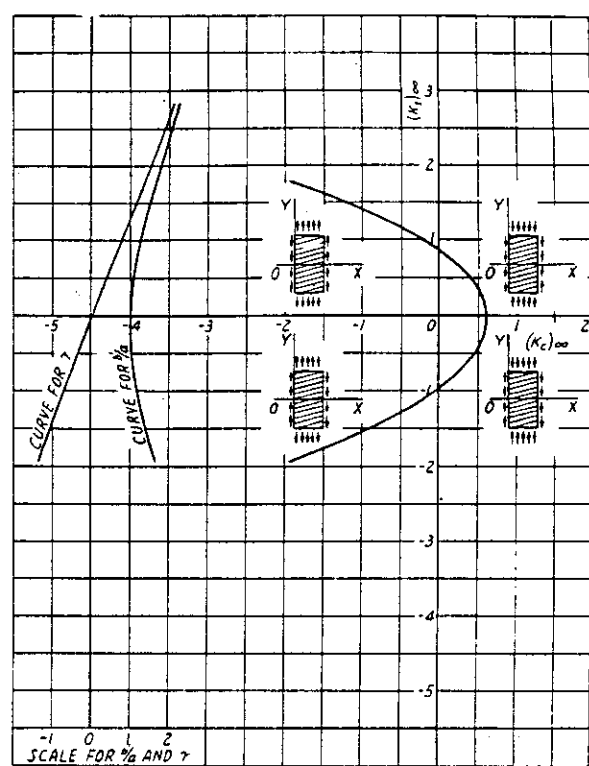


(d)
2-PLY (1:1) $\beta = 45^\circ$

Figure 2-30 (a, b, c, d). Curves of critical buckling constants for infinitely long rectangular plywood panels under combined loading. Edges simply supported. β = angle between face grain and direction of applied stress. Two-ply construction.

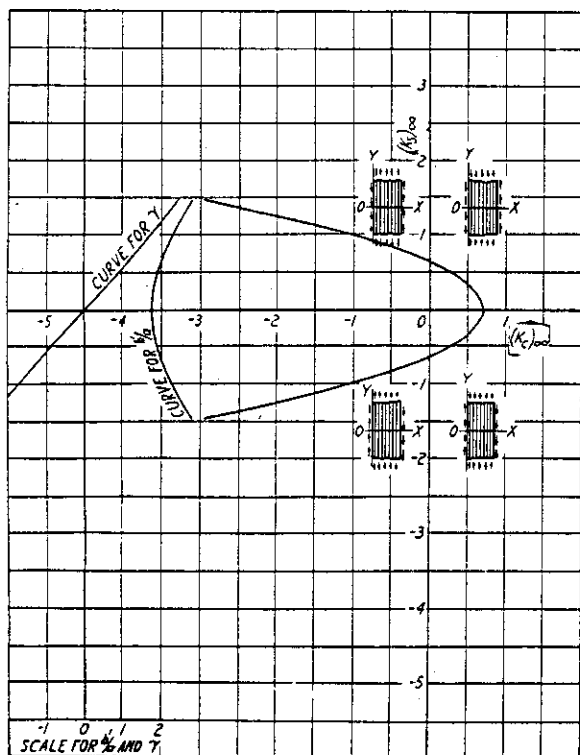


(e)
2-PLY (1:1) $\beta = 60^\circ$

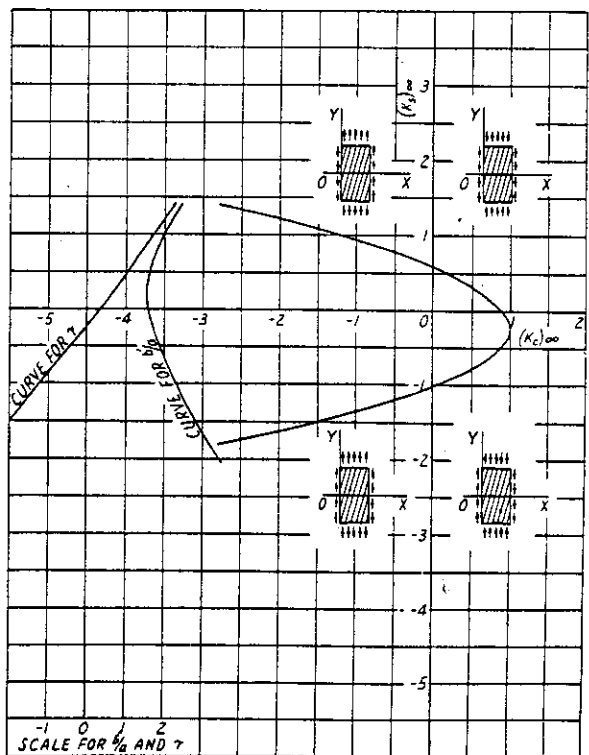


(f)
2-PLY (1:1) $\beta = 75^\circ$

Figure 2-30 (e, f)—Continued.

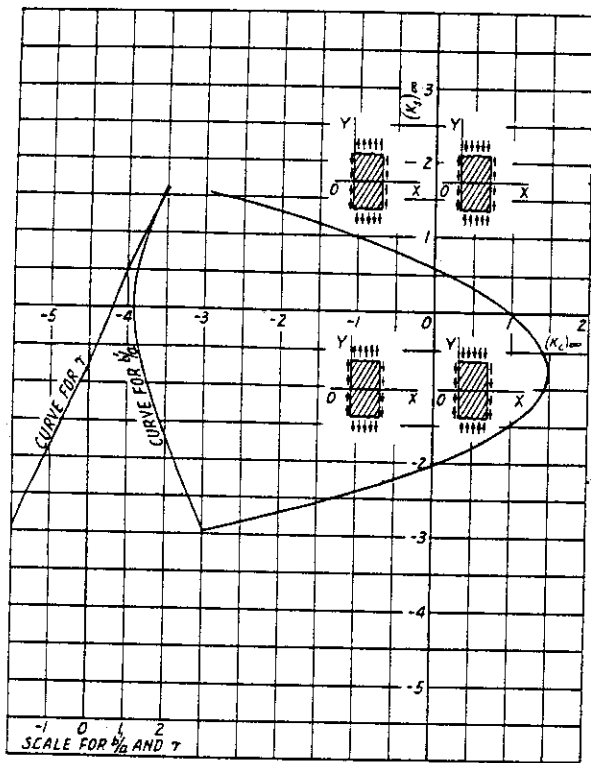


(a)
3-PLY (1:1:1) $\beta = 0^\circ$

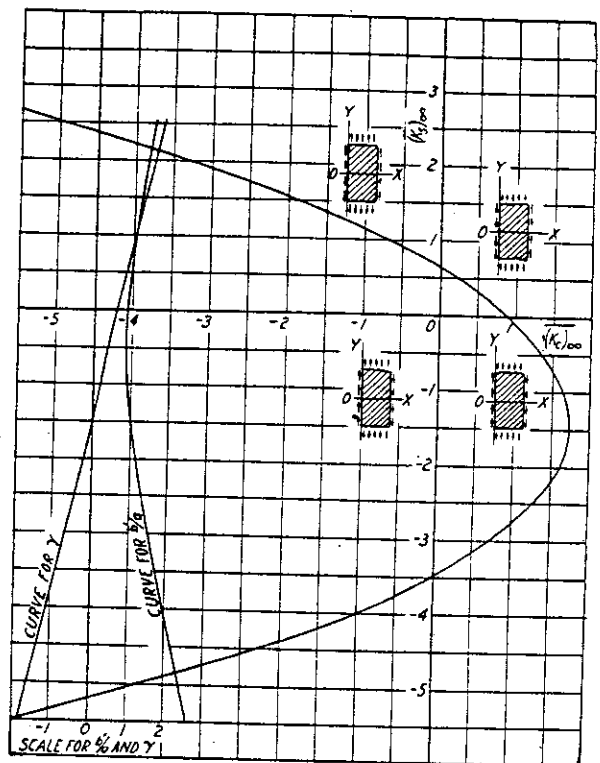


(b)
3-PLY (1:1:1) $\beta = 15^\circ$

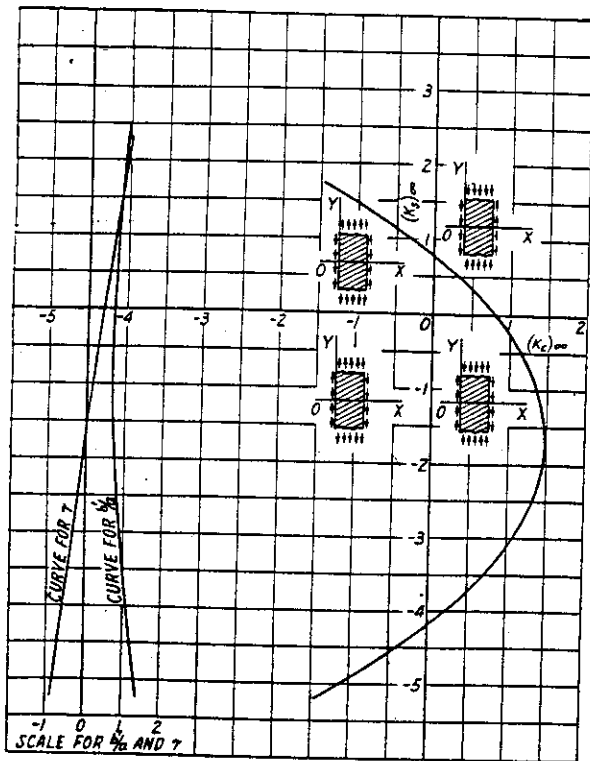
Figure 2-31 (a, b). Curves of critical buckling constants for infinitely long rectangular plywood panels under combined loading. Edges simply supported. β = angle between face grain and direction of applied stress. Three-ply construction.



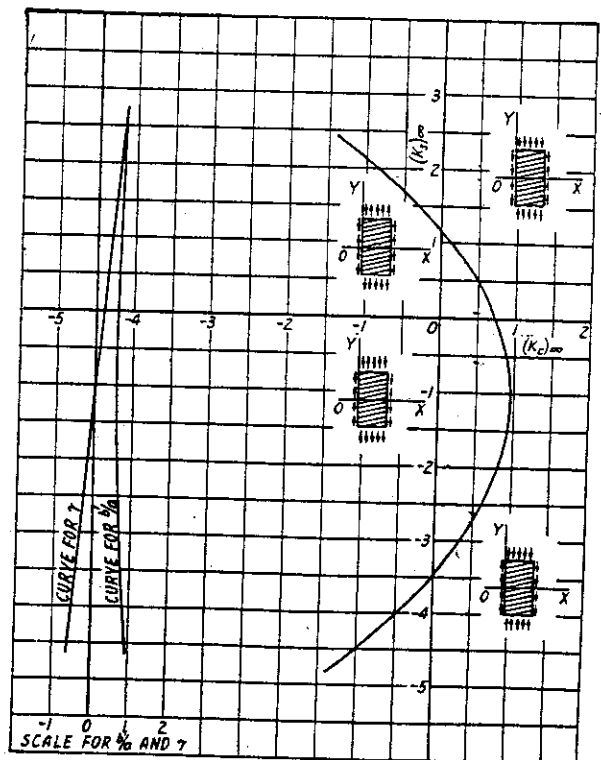
(c)
3-PLY (1:1:1) $\beta = 30^\circ$



(d)
3-PLY (1:1:1) $\beta = 45^\circ$

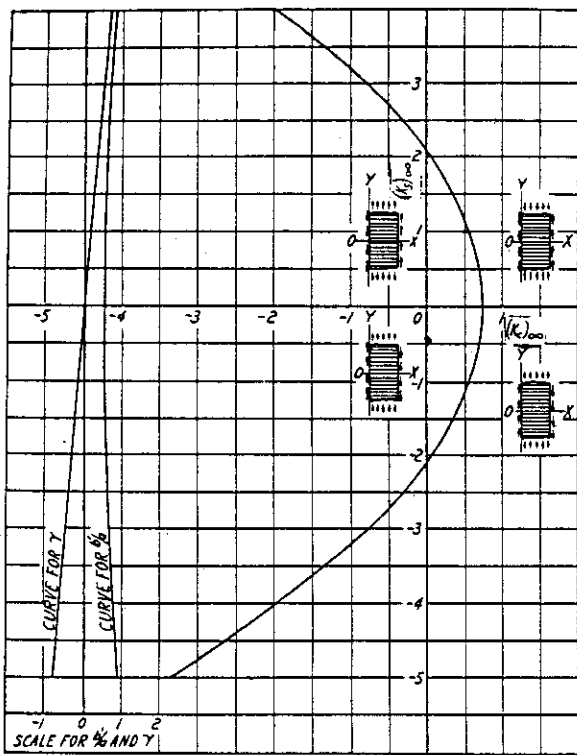


(e)
3-PLY (1:1:1) $\beta = 60^\circ$

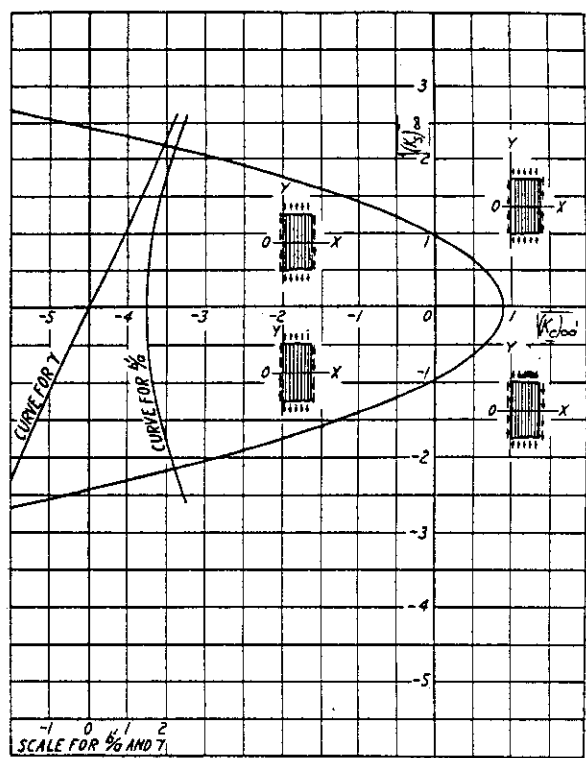


(f)
3-PLY (1:1:1) $\beta = 75^\circ$

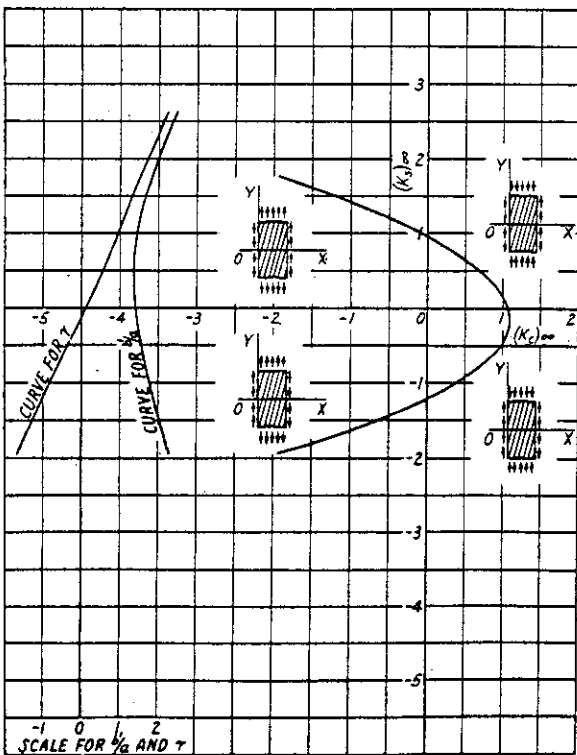
Figure 2-31 (c, d, e, f)—Continued.



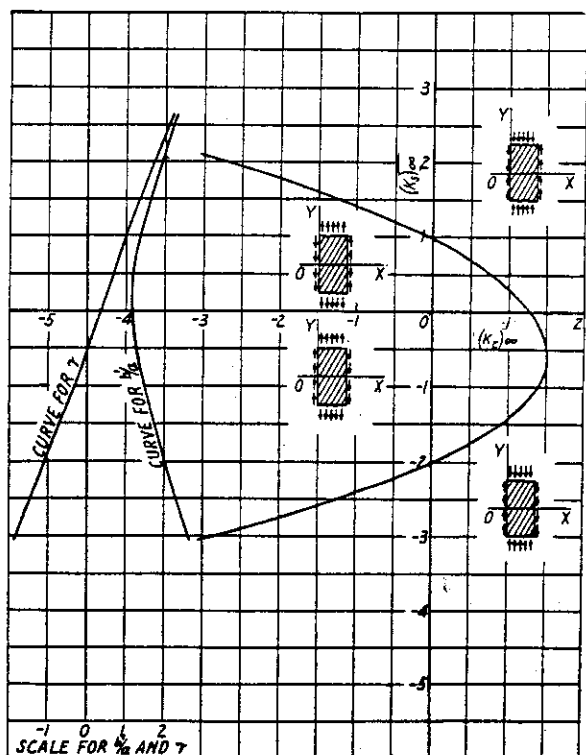
(g)
3-PLY (1:1:1) $\beta = 90^\circ$



(h)
3-PLY (1:2:1) $\beta = 0^\circ$

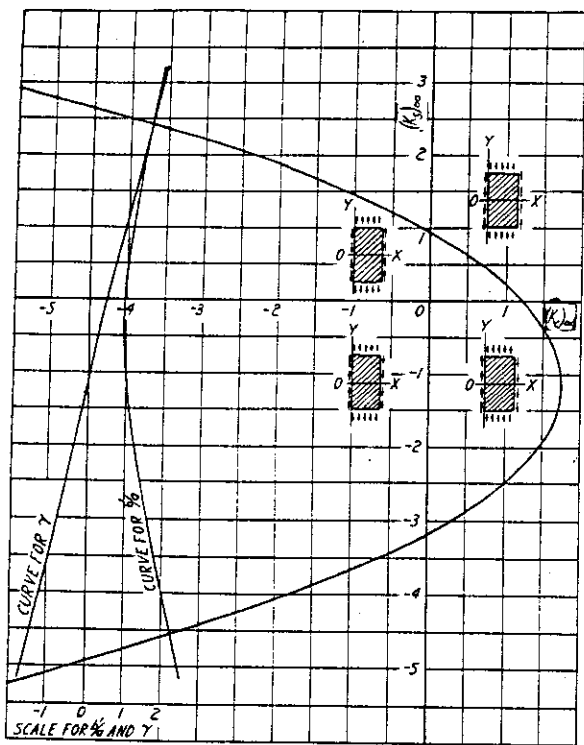


2 M 50604 F
(i)
3-PLY (1:2:1) $\beta = 15^\circ$

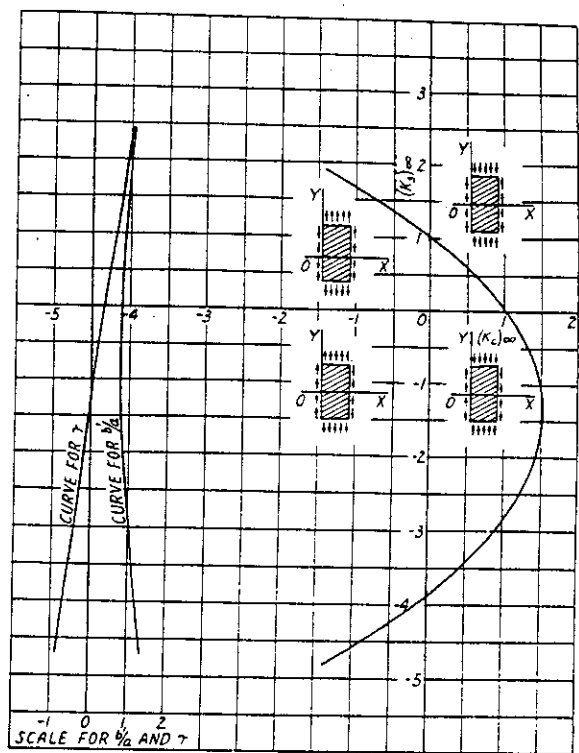


(j)
3-PLY (1:2:1) $\beta = 30^\circ$

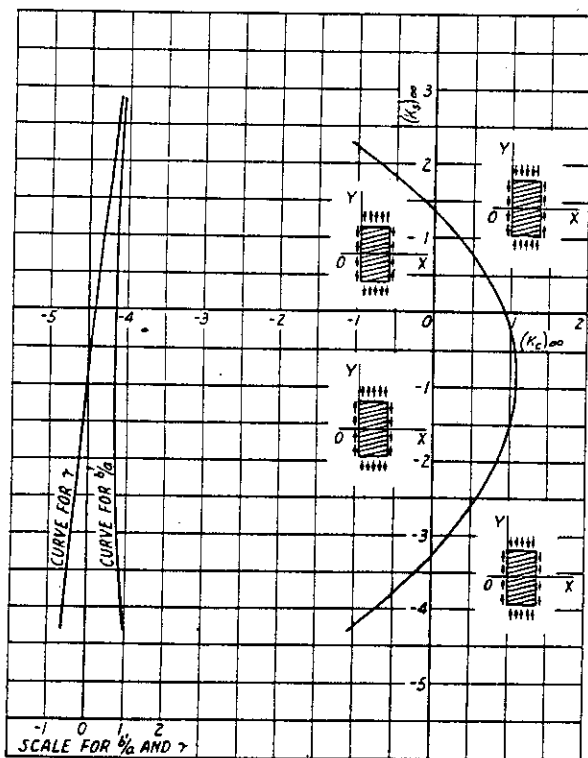
Figure 2-31 (g, h, i, j)—Continued.



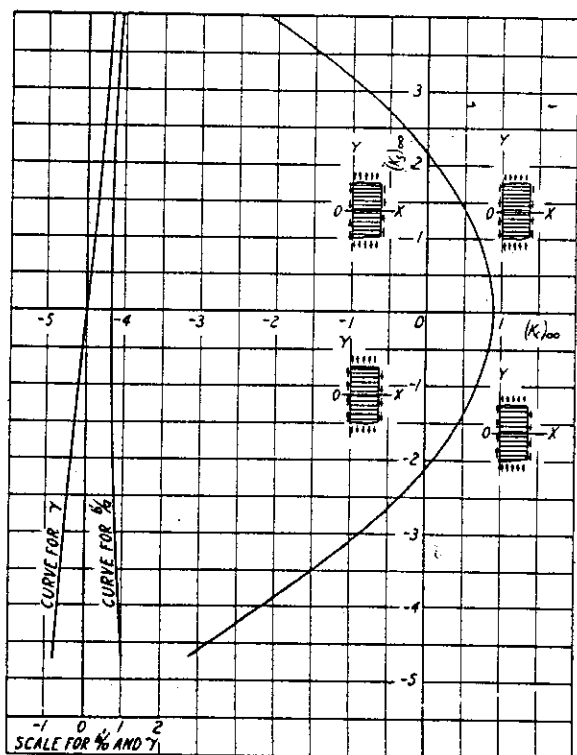
(k)
3-PLY (1:2:1) $\beta=45^\circ$



(m)
3-PLY (1:2:1) $\beta=60^\circ$

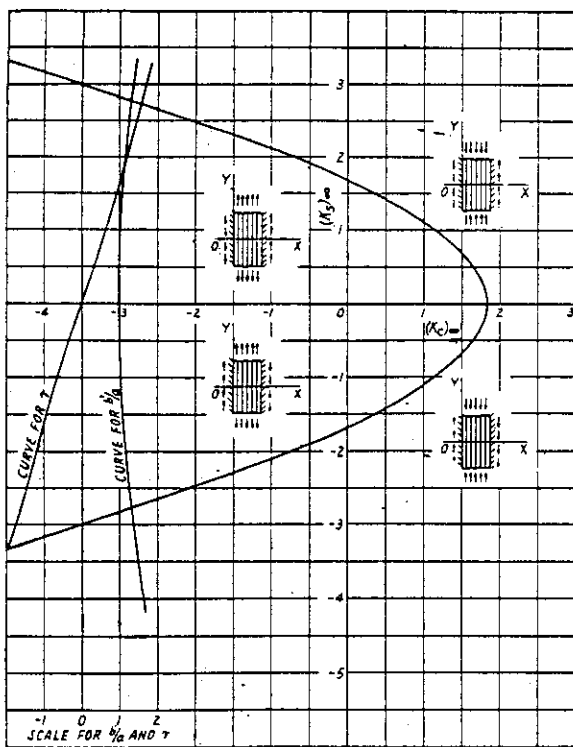


(n)
3-PLY (1:2:1) $\beta=75^\circ$

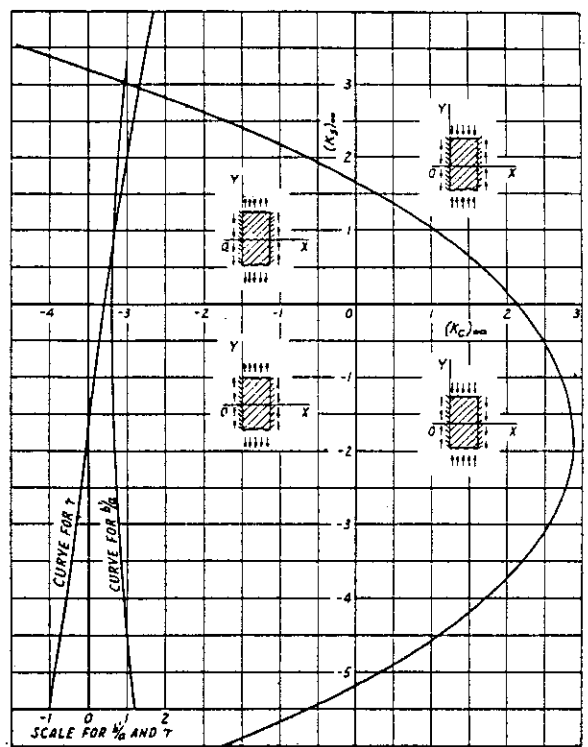


(p)
3-PLY (1:2:1) $\beta=90^\circ$

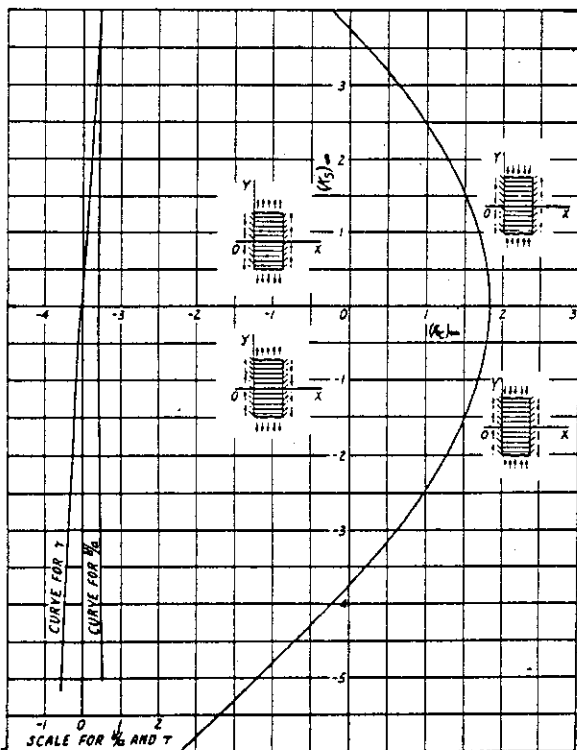
Figure 2-81 (k, m, n, p)—Continued.



(a)
3-PLY (1:2:1) $\beta = 0^\circ$

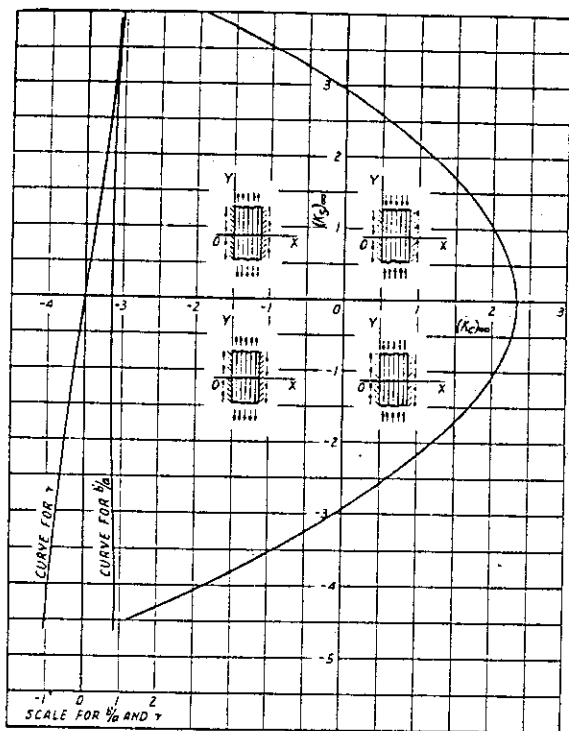


(b)
3-PLY (1:2:1) $\beta = 45^\circ$

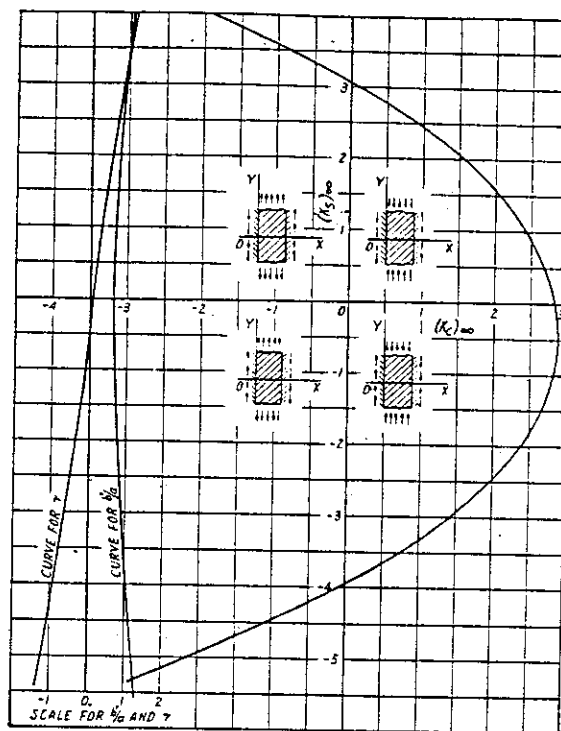


(c)
FIG. 37
3-PLY (1:2:1) $\beta = 90^\circ$

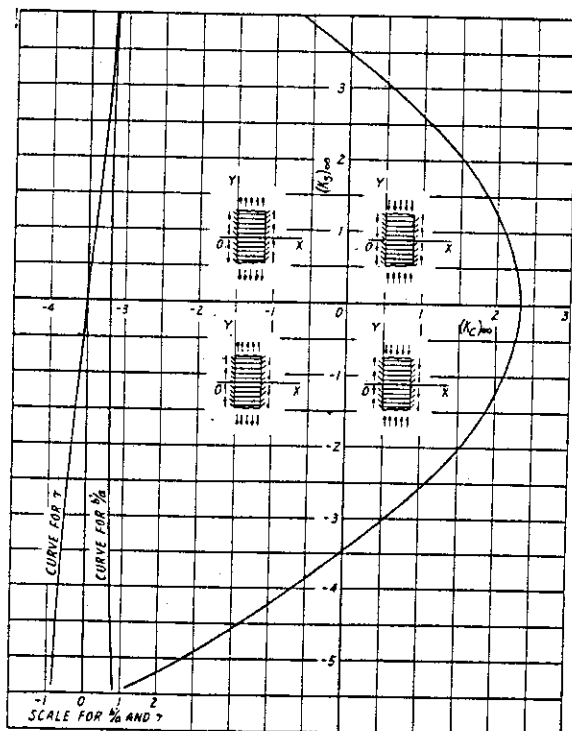
Figure 2-32 (a, b, c). Curves of critical buckling constants for infinitely long rectangular plywood panels under combined loading with edges clamped. β = angle between face grain and direction of applied stress. Three-ply construction.



(d)
5-PLY (1:2:2:2:1) $\beta = 0^\circ$

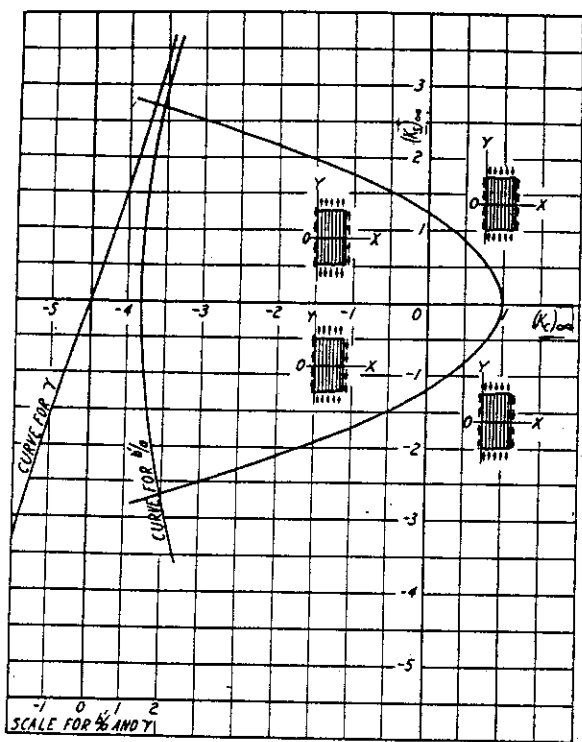


(e)
6-PLY (1:2:2:2:2:1) $\beta = 45^\circ$

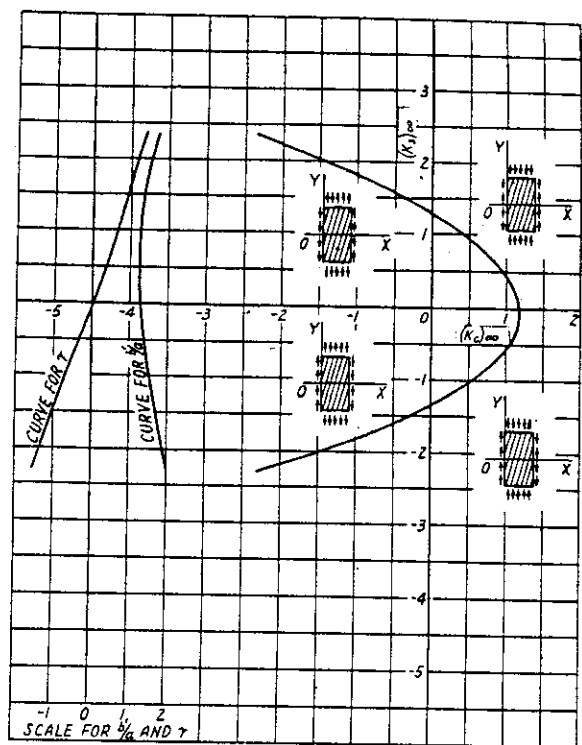


(f)
5-PLY (1:2:2:2:1) $\beta = 90^\circ$

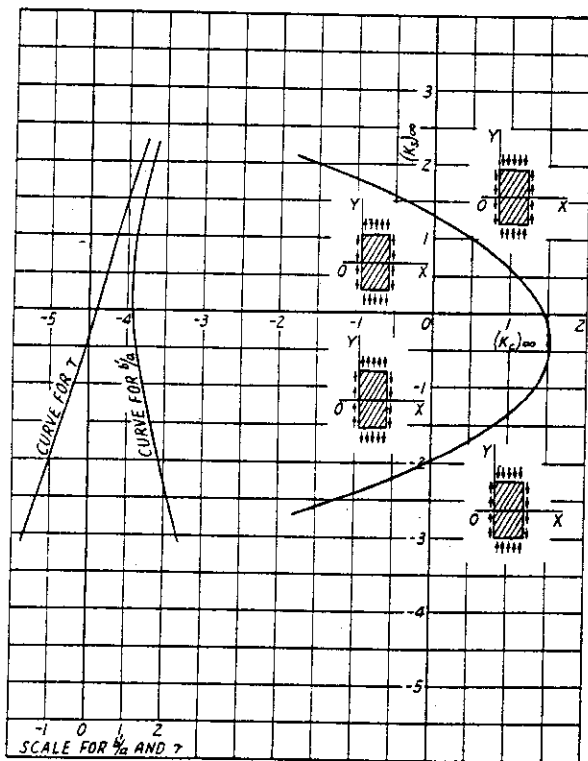
Figure 2-33 (d, e, f). Curves of critical buckling constants for infinitely long rectangular plywood panels under combined loading with edges clamped. β = angle between face grain and direction of applied stress. Five-ply construction.



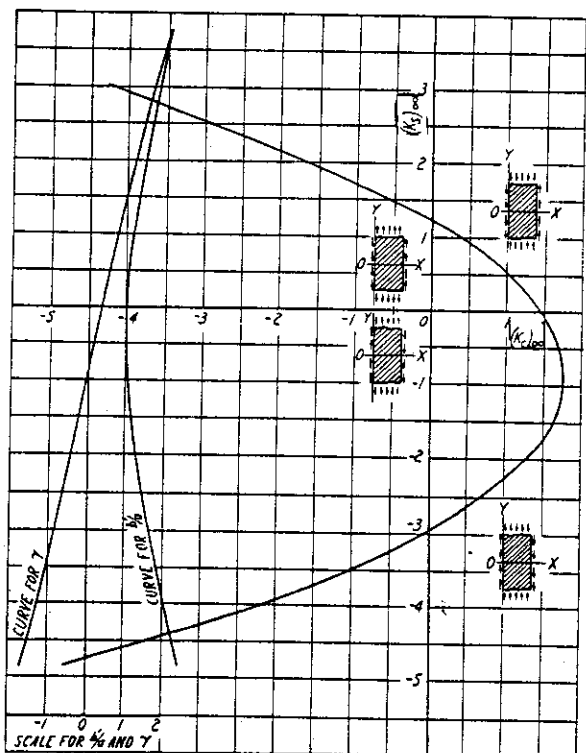
(a)
5-PLY (1:1:1:1:1) $\beta = 0^\circ$



(b)
5-PLY (1:1:1:1:1) $\beta = 15^\circ$

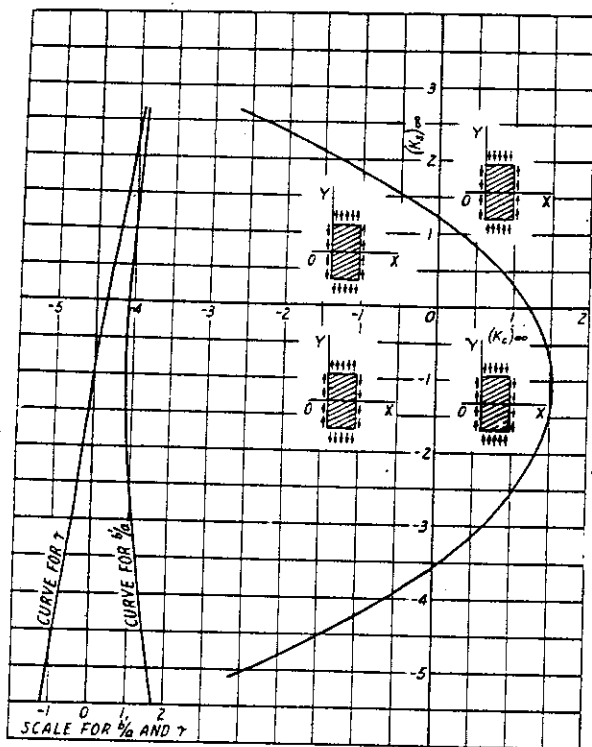


(c)
5-PLY (1:1:1:1:1) $\beta = 30^\circ$

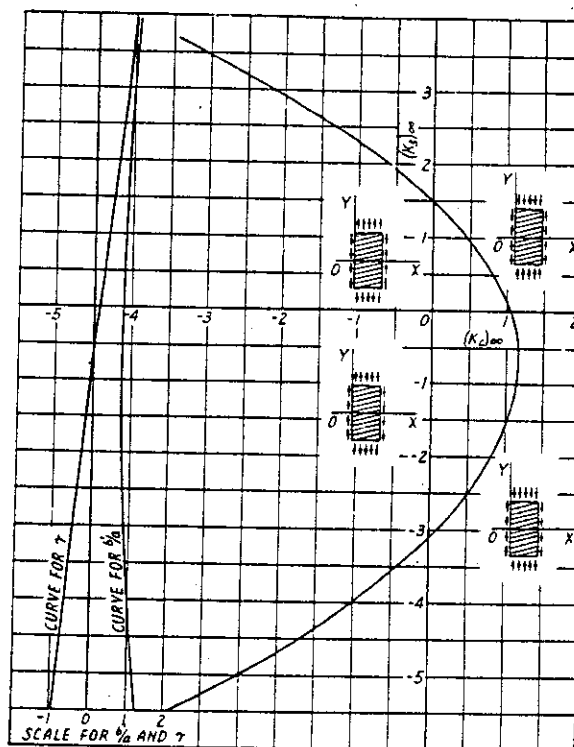


(d)
5-PLY (1:1:1:1:1) $\beta = 45^\circ$

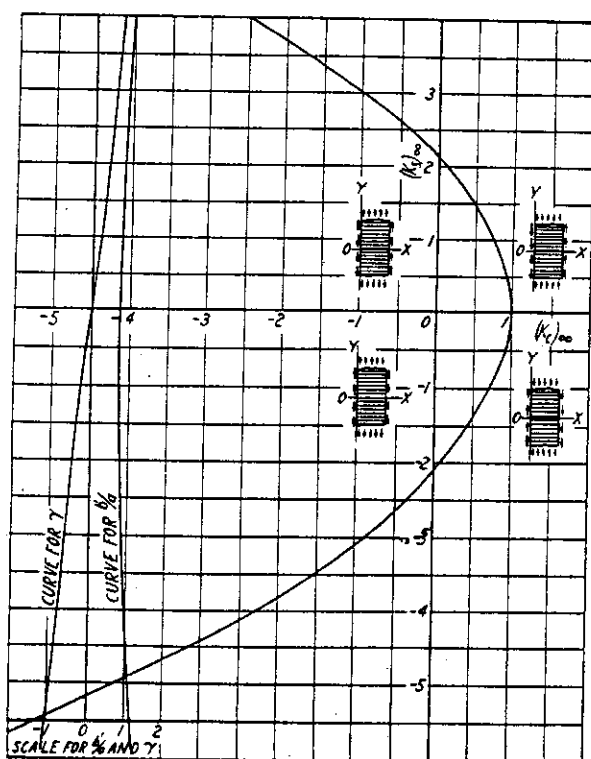
Figure 2-34 (a, b, c, d). Curves of critical buckling constants for infinitely long rectangular plywood panels under combined loading. Edges simply supported. β = angle between face grain and direction of applied stress. Five-ply construction.



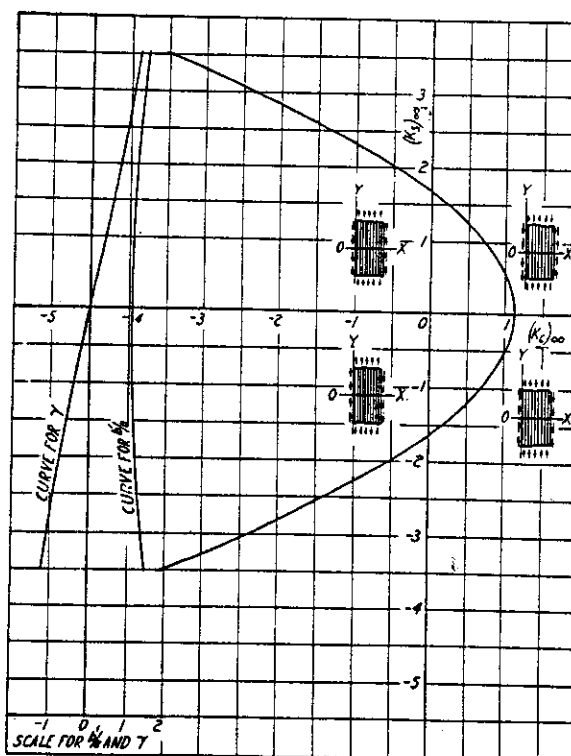
(e)
5-PLY (1:1:1:1:1) $\beta = 60^\circ$



(f)
5-PLY (1:1:1:1:1) $\beta = 75^\circ$

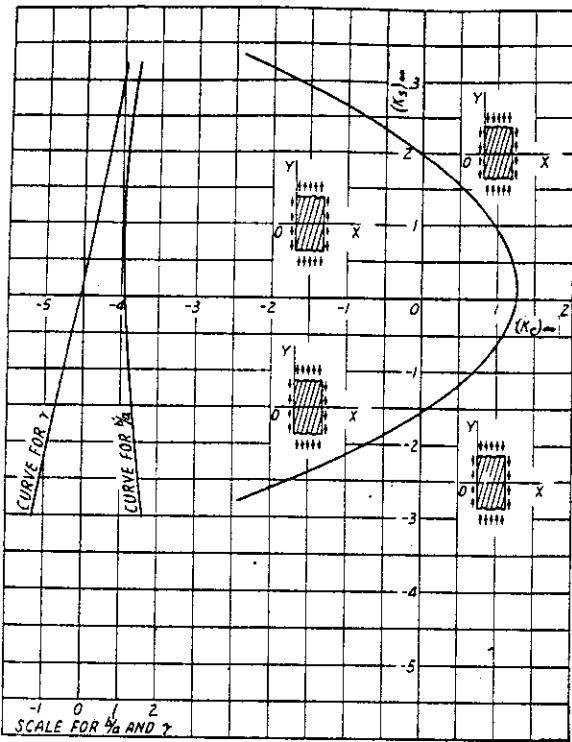


(g)
5-PLY (1:1:1:1:1) $\beta = 90^\circ$

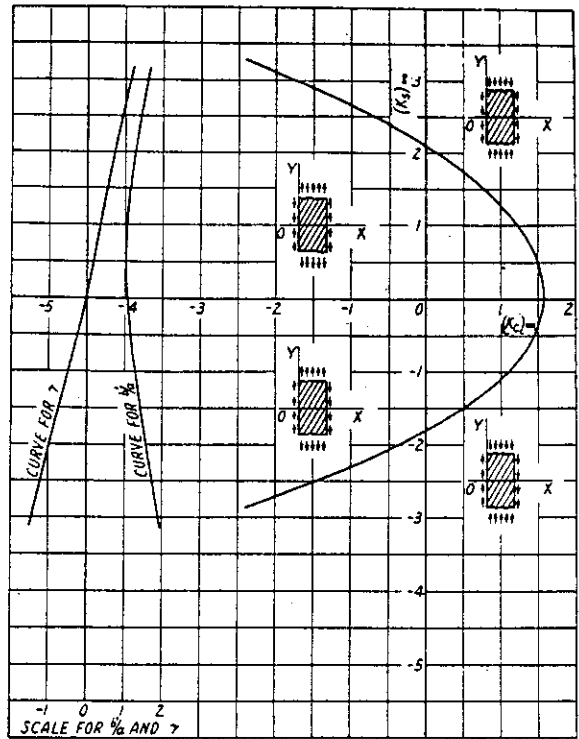


(h)
5-PLY (1:2:2:2:1) $\beta = 0^\circ$

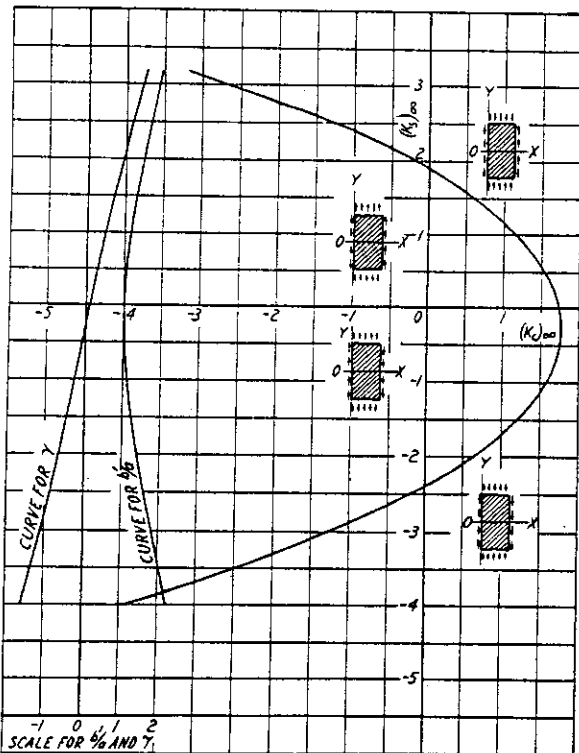
Figure 2-84 (e, f, g, h)—Continued.



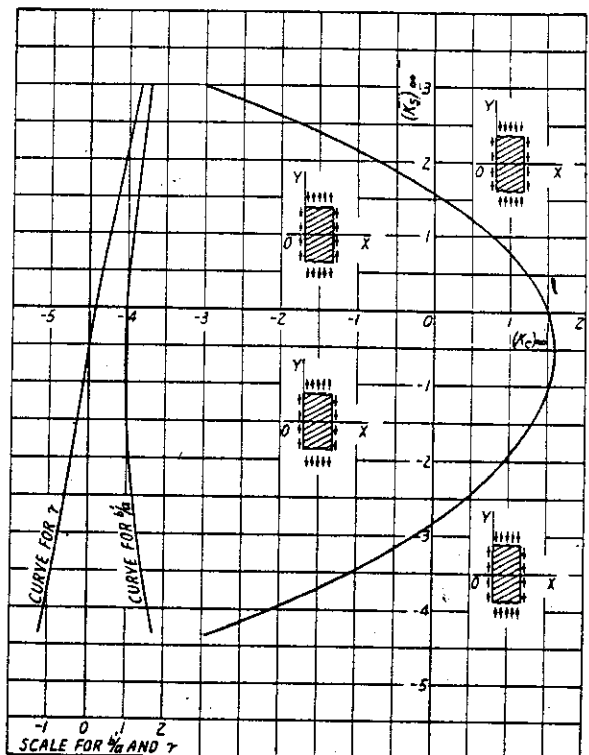
(i)
5-PLY (1:2:2:2:1) $\beta = 15^\circ$



(j)
5-PLY (1:2:2:2:1) $\beta = 30^\circ$

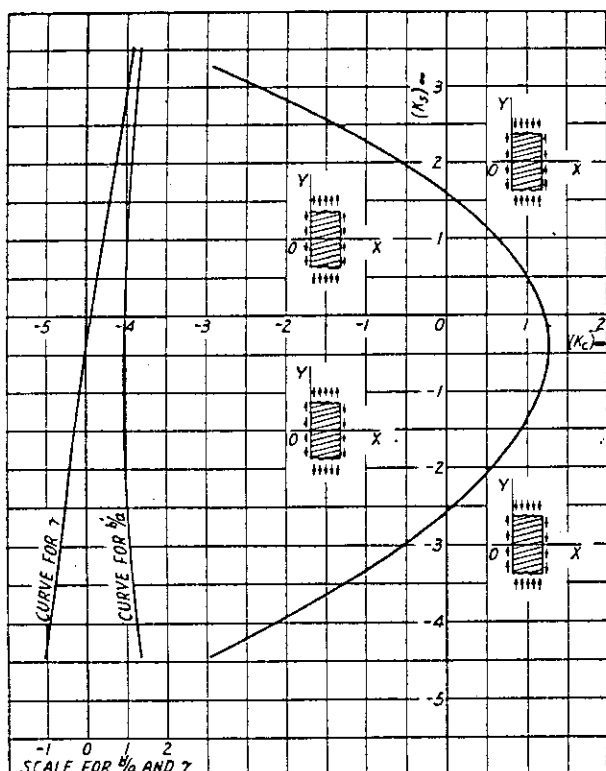


(k)
5-PLY (1:2:2:2:1) $\beta = 45^\circ$

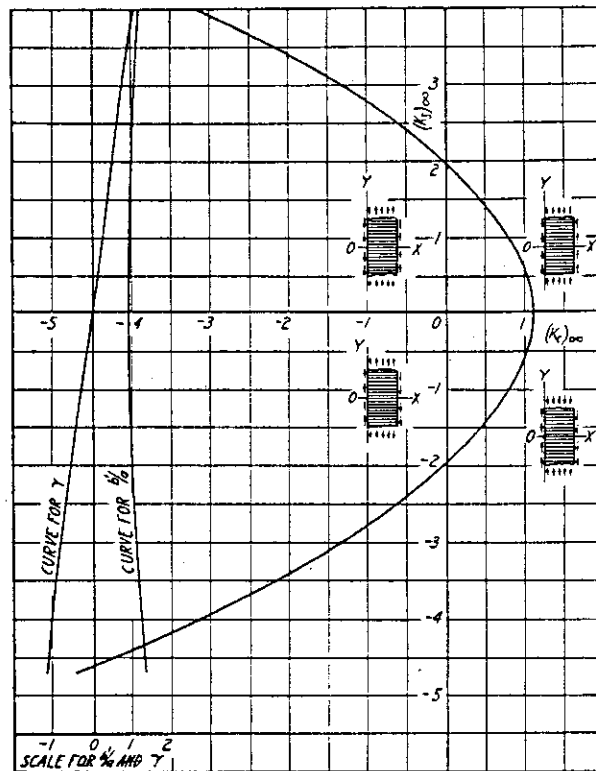


(m)
5-PLY (1:2:2:2:1) $\beta = 60^\circ$

Figure 2-34 (i, j, k, m)—Continued.



(n)
5-PLY (1:2:2:2:1) $\beta = 75^\circ$



(p)
5-PLY (1:2:2:2:1) $\beta = 90^\circ$

Figure 2-34 (n, p)—Continued.

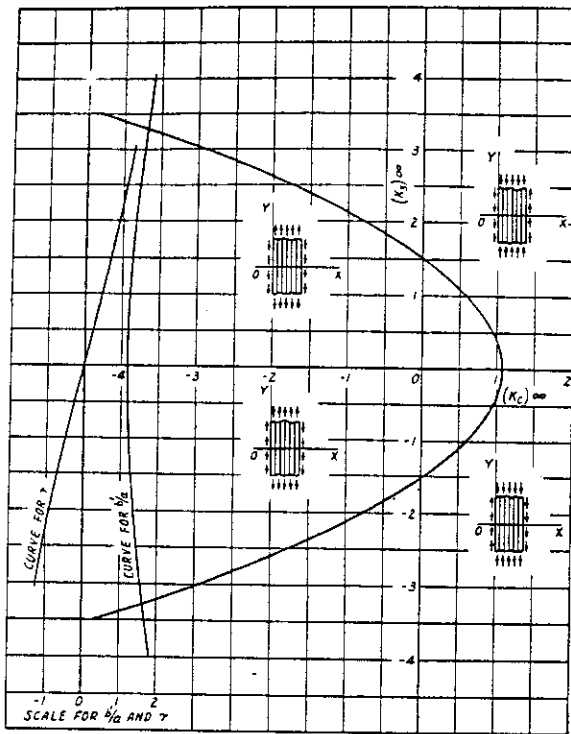
recting $K_{s,\infty}$ values or $K_{c,\infty}$ values in table 2-14 for panel size by means of figure 2-37. In using these figures b'/a is first obtained from table 2-14 and b/b' computed. For a more exact determination of K_s or K_c or to determine these buckling constants for a plywood construction different from those specified in AN-NN-P-511b or figures 2-30 to 2-36, calculate $\frac{E_{fw}}{E_{fw} + E_{fx}}$ in accordance with section 2.52, read $K_{s,\infty}$ or $K_{c,\infty}$ and b'/a from figures 2-38 to 2-41 and correct for panel size by means of figure 2-37.

2.7151. *Combined compression (or tension) and shear.* The analytical method of determining the critical buckling stresses for rectangular panels subjected to combined loadings is quite complicated, and only the graphical solutions for a few types of plywood construction are given in figures 2-30 to 2-36.

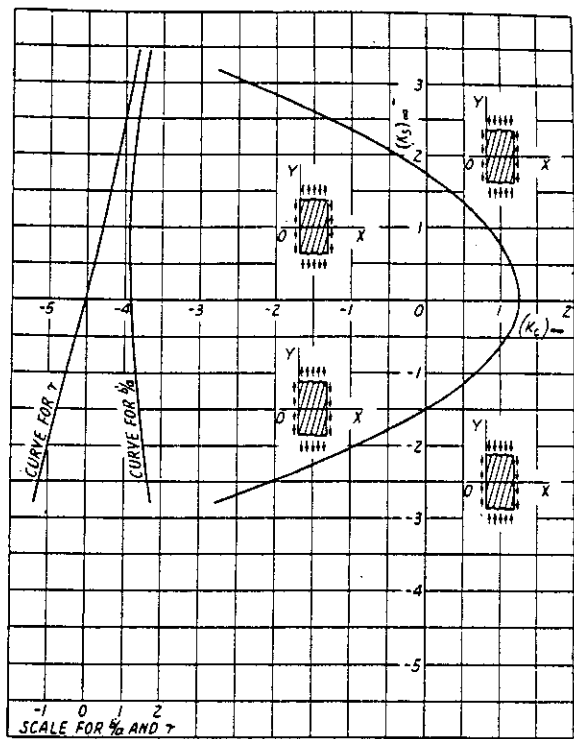
When the plywood construction being used is not the same as any of those illustrated, its buckling constants may be obtained by a straight line interpolation (or extrapolation), on the basis

of $\frac{E_{fw}}{E_{fw} + E_{fx}}$, of the buckling constants for two plywood constructions whose values of the ratio $\frac{E_{fw}}{E_{fw} + E_{fx}}$ are fairly close to that of the plywood under consideration. The values of these ratios for the plywood constructions considered in figures 2-30 to 2-36 may be calculated with sufficient accuracy by assuming $E_r = 0.05 E_L$.

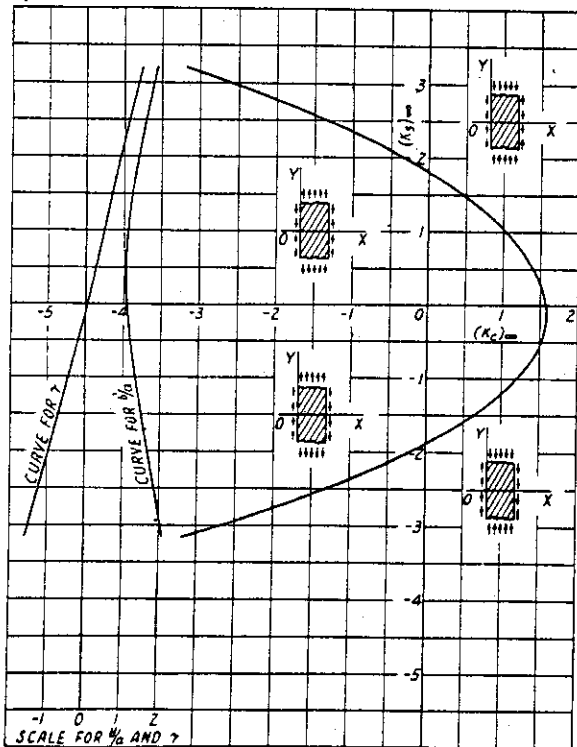
These figures apply to panels of infinite length and values of the buckling constants from the curves must be corrected for actual panel length. Values of the shear constant $K_{s,\infty}$ and the compression constant $K_{c,\infty}$ are indicated on the vertical and horizontal axes, respectively. The points at which the curve crosses these axes give the values of $K_{s,\infty}$ or $K_{c,\infty}$ at which buckling will just occur in a panel of infinite length in either pure shear or pure compression. The particular combination of stresses represented by each of the four quadrants is shown by the small stress sketches. Buckling will occur under these combined stresses whenever the location of a point $K_{s,\infty}$, $K_{c,\infty}$, lies on or outside the curve.



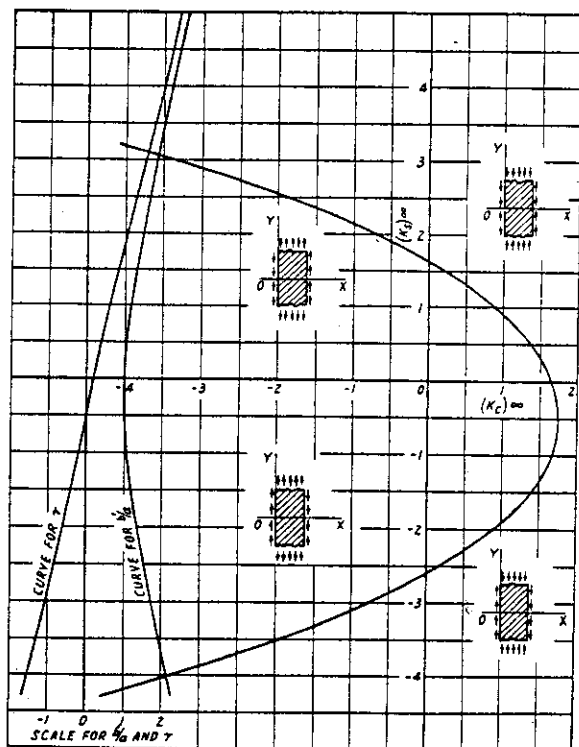
(a)
9-PLY (|||||) $\beta = 0^\circ$



(b)
9-PLY (|||||) $\beta = 15^\circ$

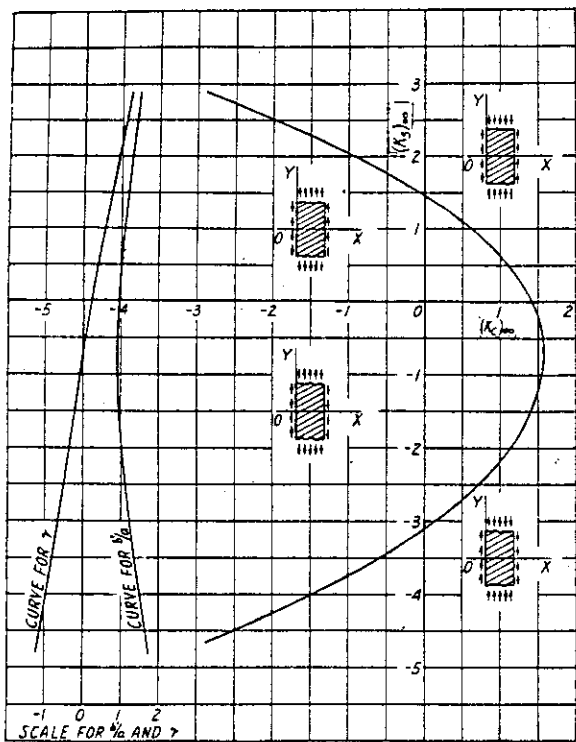


(c)
9-PLY (|||||) $\beta = 30^\circ$

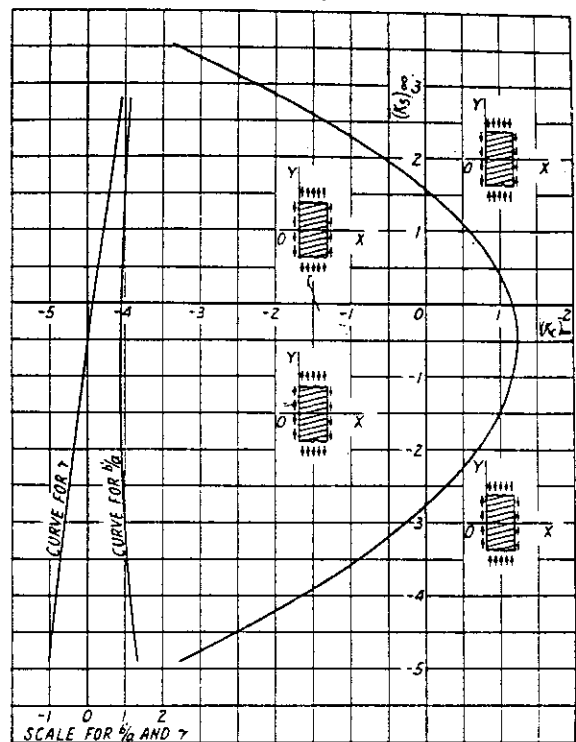


(d)
9-PLY (|||||) $\beta = 45^\circ$

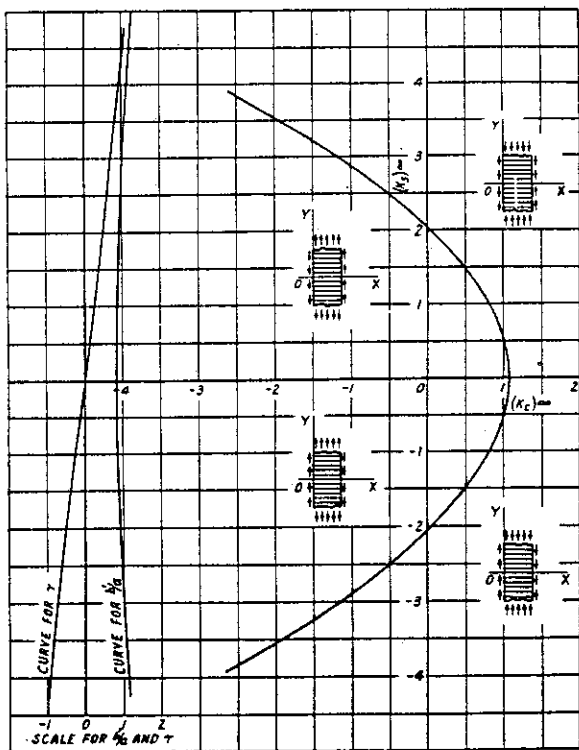
Figure 2-35 (a, b, c, d). Curves of critical buckling constants for infinitely long rectangular plywood panels under combined loading. Edges simply supported. β = angle between face grain and direction of applied stress. Nine-ply construction.



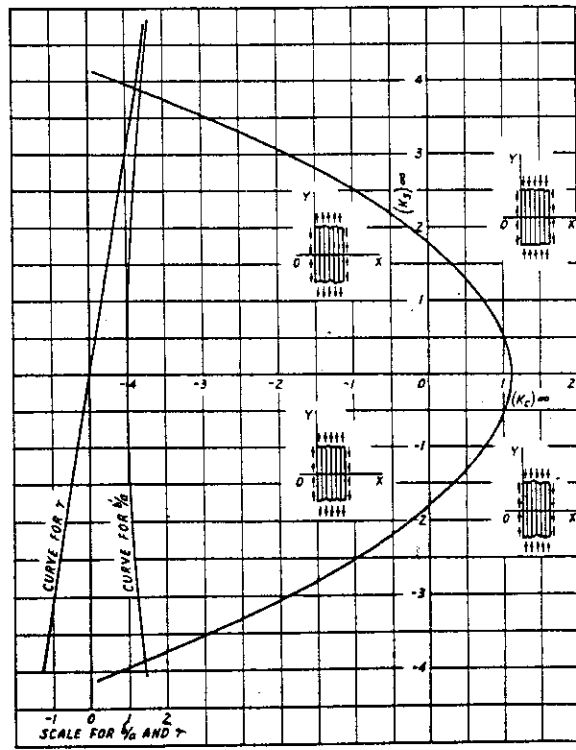
(e)
9-PLY (1:1:1:1:1:1:1:1:1) $\beta = 60^\circ$



(f)
9-PLY (1:1:1:1:1:1:1:1:1) $\beta = 75^\circ$

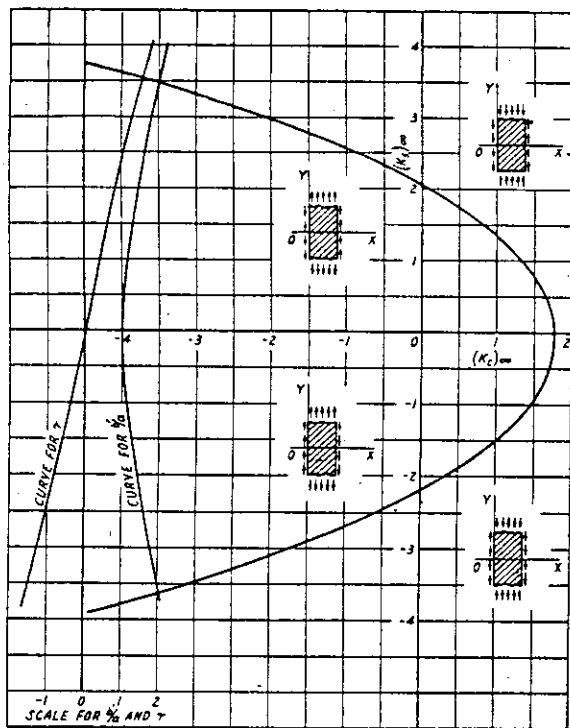


Z M 50613 3
(g)
9-PLY (1:1:1:1:1:1:1:1:1) $\beta = 90^\circ$

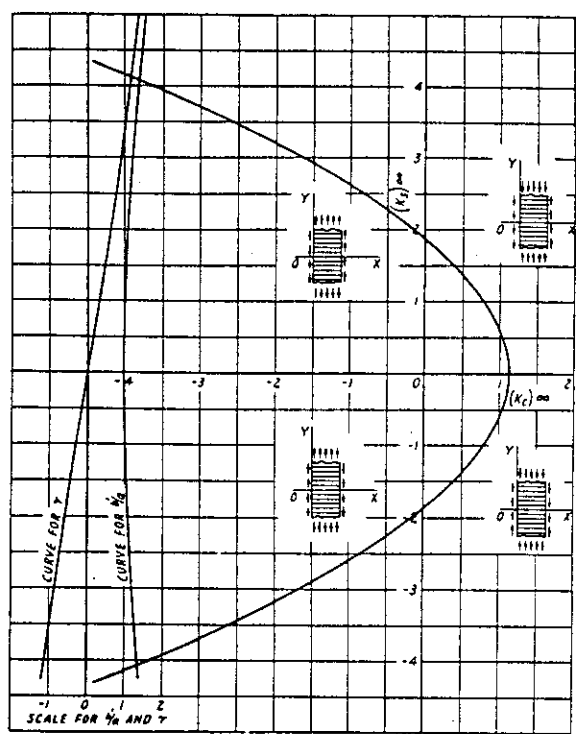


(h)
9-PLY (1:2:2:2:2:2:2:2:1) $\beta = 0^\circ$

Figure 2-35 (e, f, g, h)—Continued.

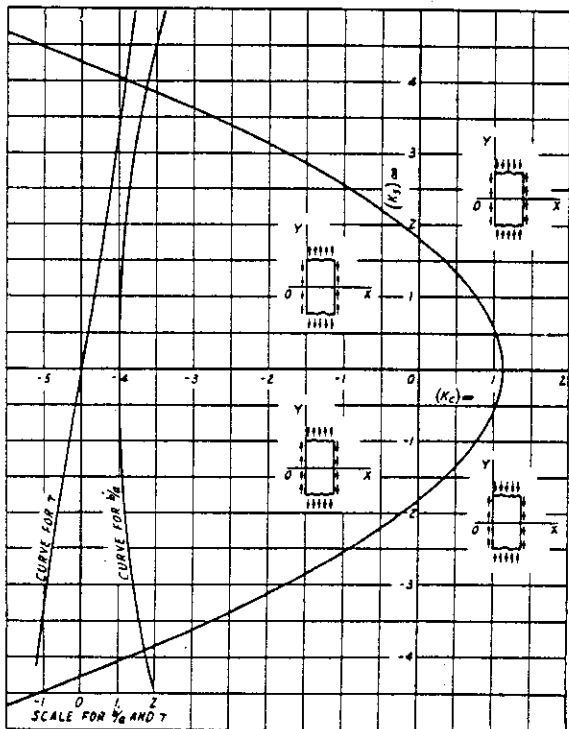


(i)
9-PLY (1:2:2:2:2:2:2:2:2) $\beta = 45^\circ$

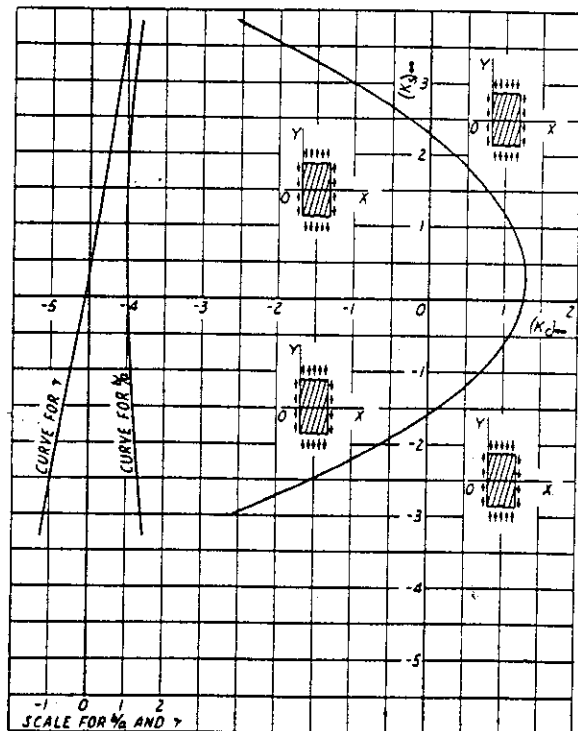


(j)
9-PLY (1:2:2:2:2:2:2:2:2) $\beta = 90^\circ$

Figure 2-35 (i, j)—Continued.

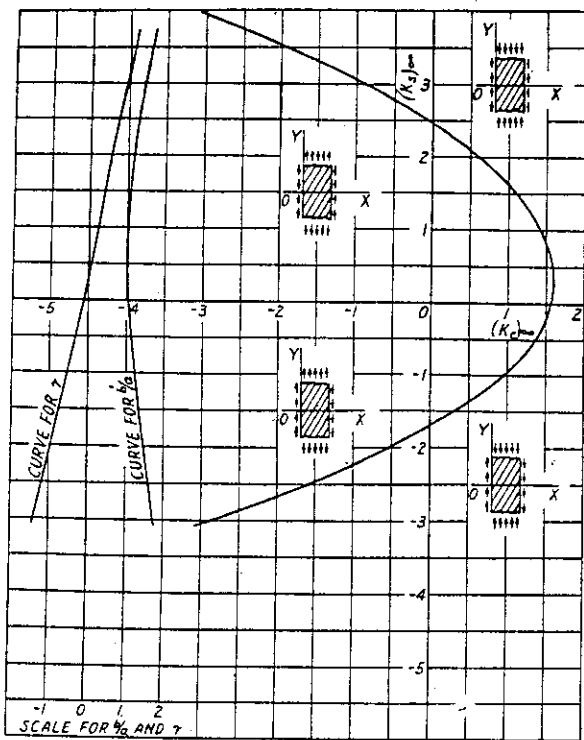


(a)
 ∞ -PLY $\beta = 0^\circ$ AND $\beta = 90^\circ$

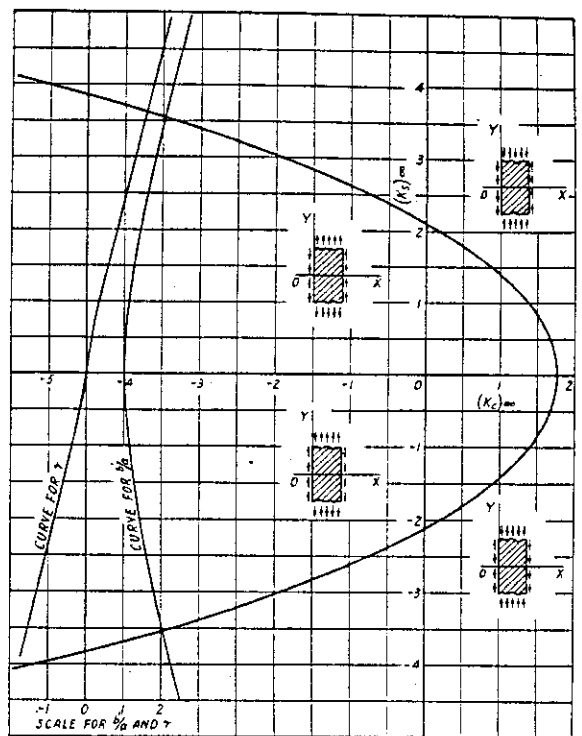


(b)
 ∞ -PLY $\beta = 15^\circ$

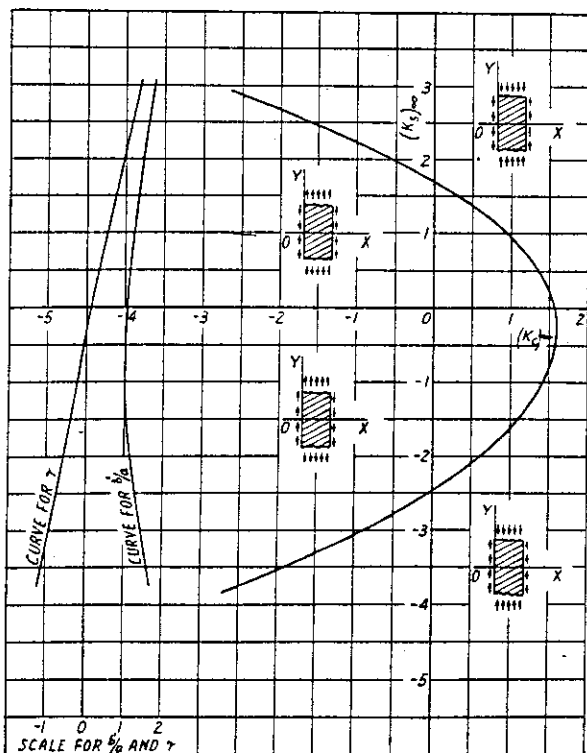
Figure 2-36 (a, b). Curves of critical buckling constants for infinitely long rectangular plywood panels under combined loading. Edges simply supported. β = angle between face grain and direction of applied stress. Infinite-ply construction.



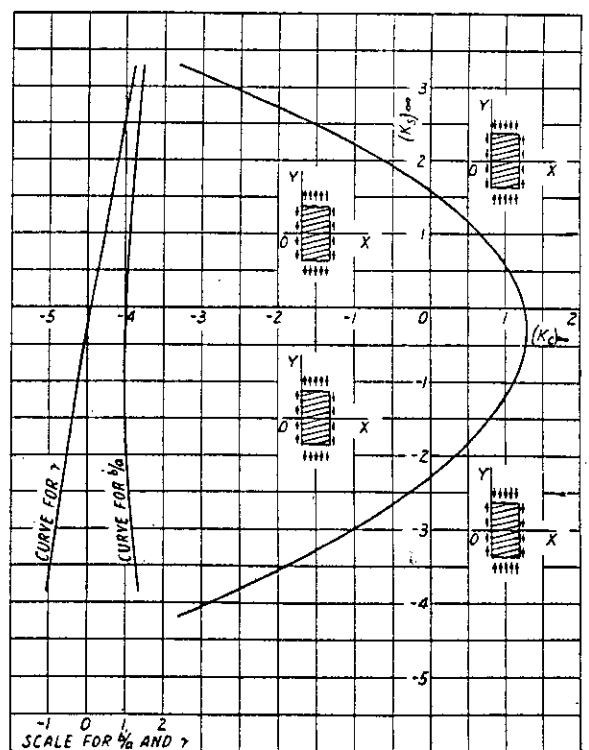
(c)
 ∞ -PLY $\beta = 30^\circ$



(d)
 ∞ -PLY $\beta = 45^\circ$



(e)
 ∞ -PLY $\beta = 60^\circ$



(f)
 ∞ -PLY $\beta = 75^\circ$

Figure 2-36 (c, d, e, f)—Continued.

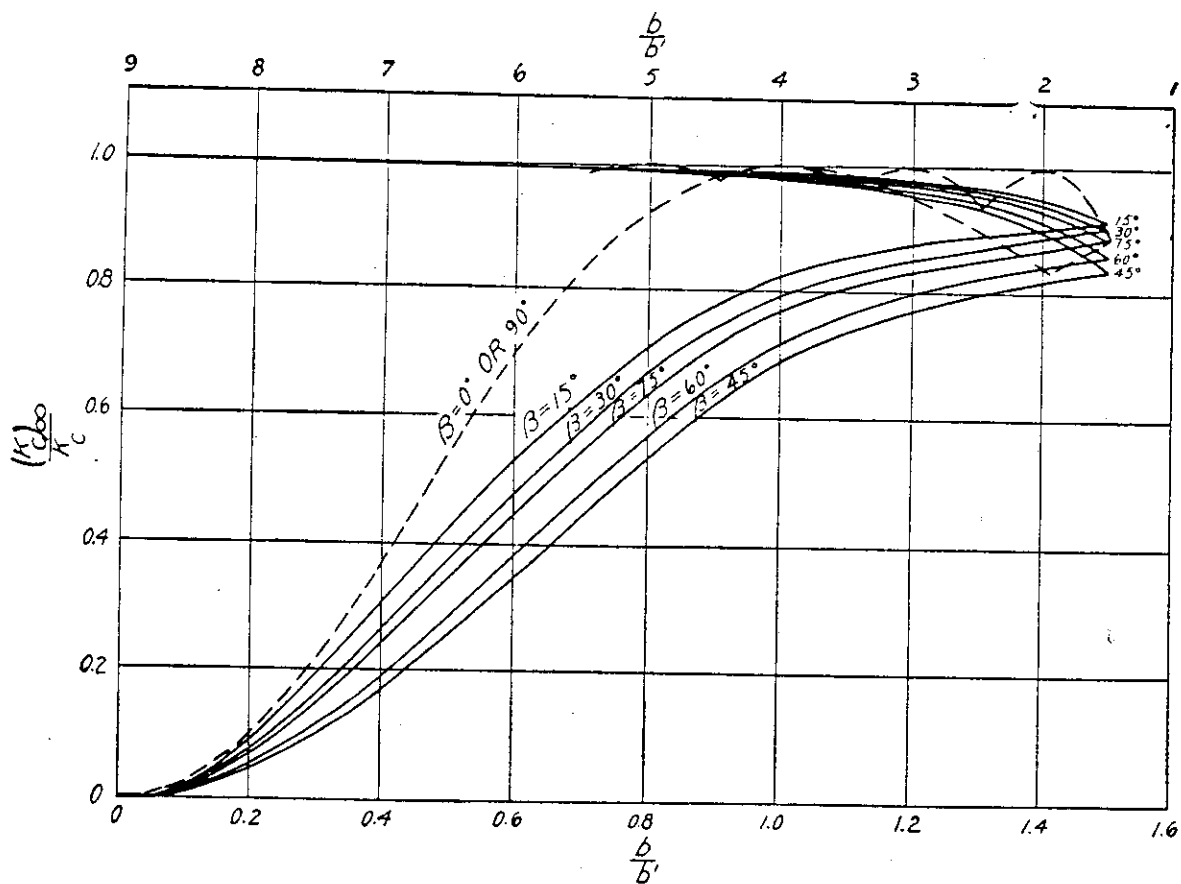
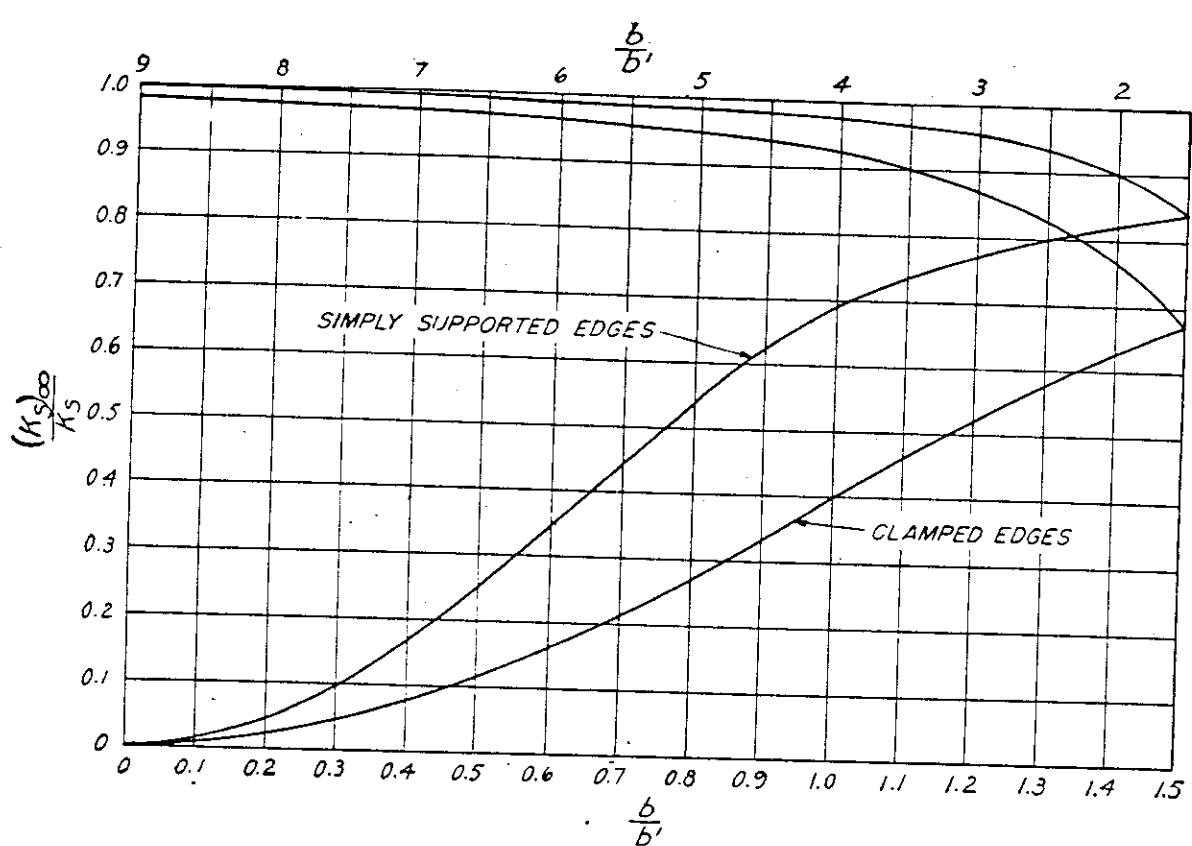


Figure 2-37. Corrections for panel size: Top, when $\beta = 0^\circ$, 45° , or 90° and panel is subjected to shear stress; bottom, when the panel is subjected to compression with the edges simply supported and $\beta = 0^\circ$ or 90° is a computed curve (ref. 2-50).

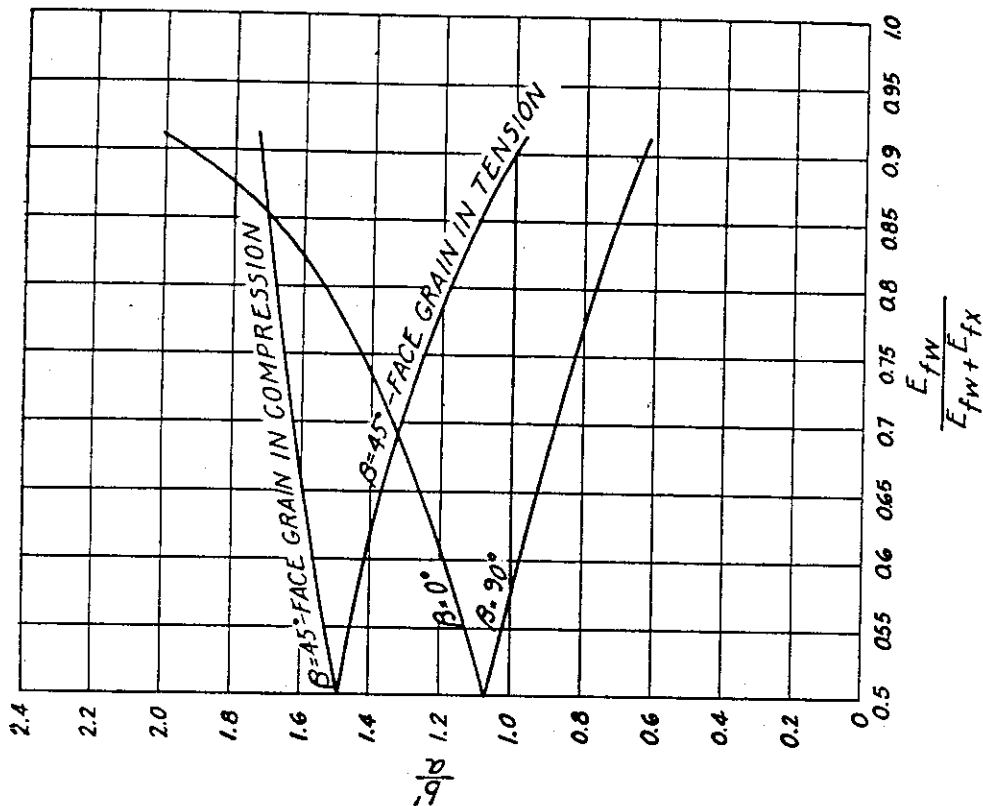
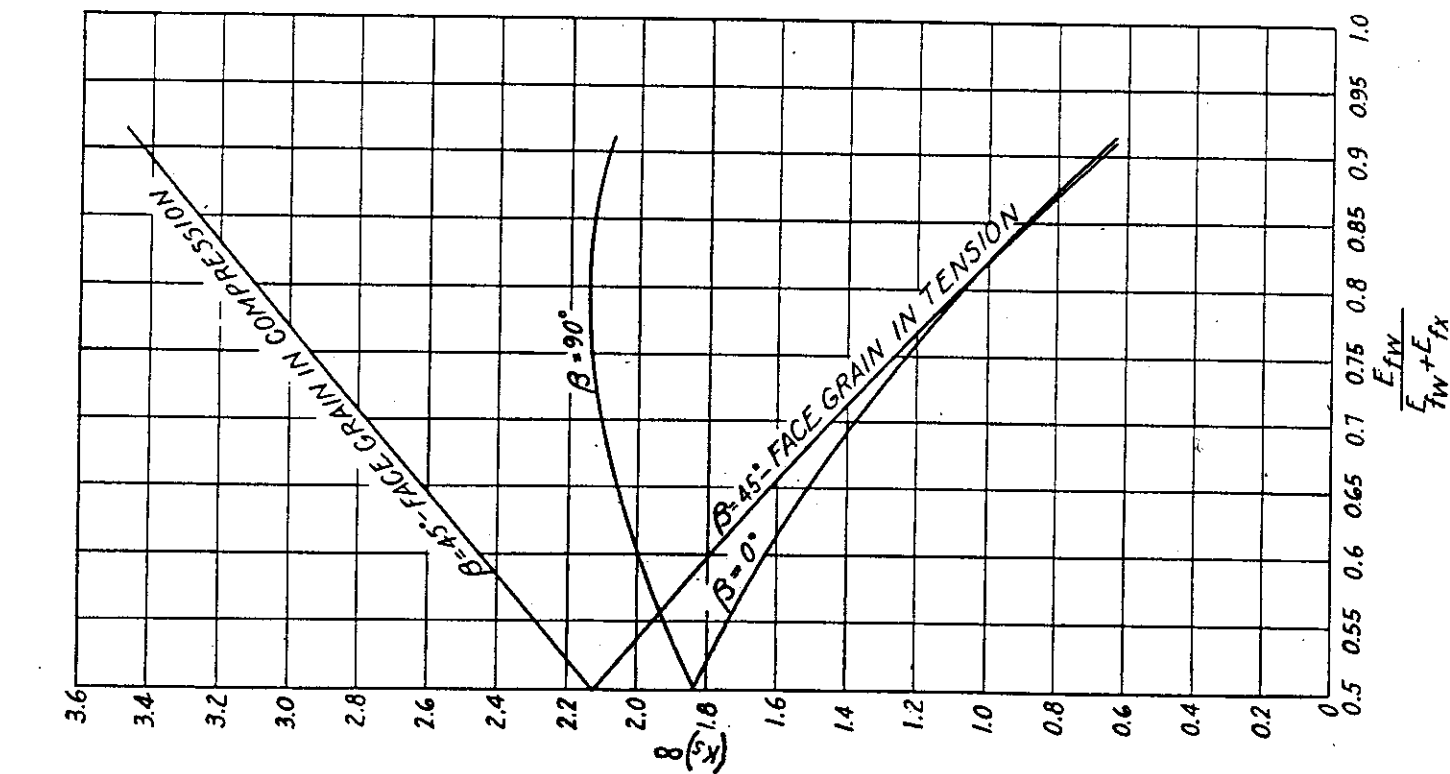


Figure 2-38 (left). Buckling of infinitely long plates of symmetrical construction under uniform shear for determination of $(K_s)_\infty$. Edges simply supported.

Figure 2-39. Buckling of infinitely long plates of symmetrical construction under uniform shear for determination of b'/a . Edges simply supported.

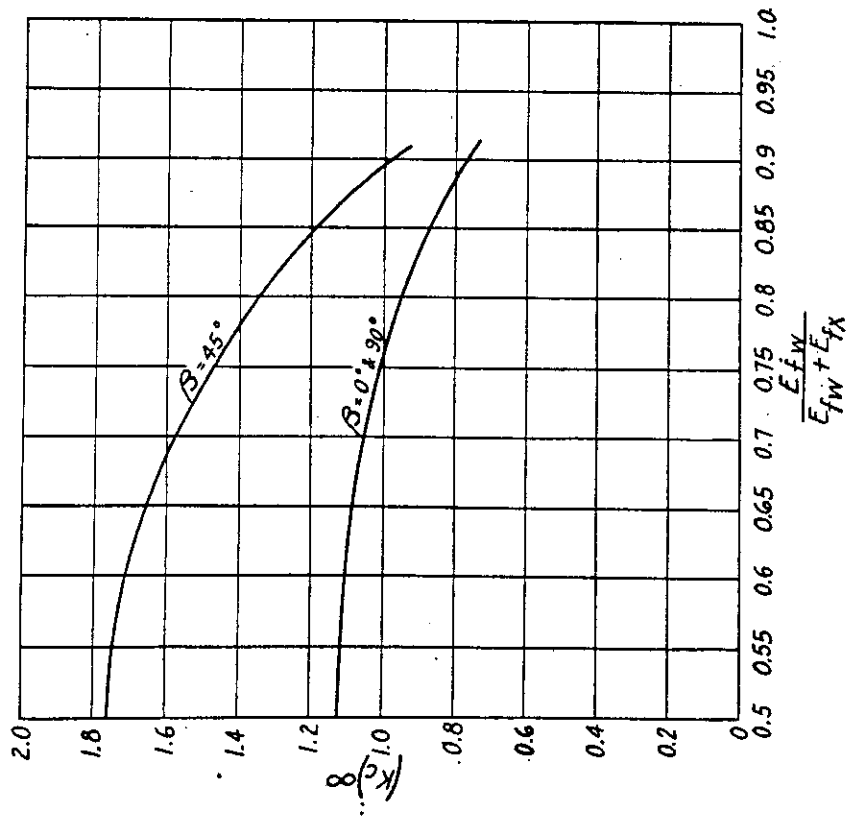


Figure 2-40. Buckling of infinitely long plates of symmetrical construction under uniform compression for determination of $(K_c)_{\infty}$. Edges simply supported.

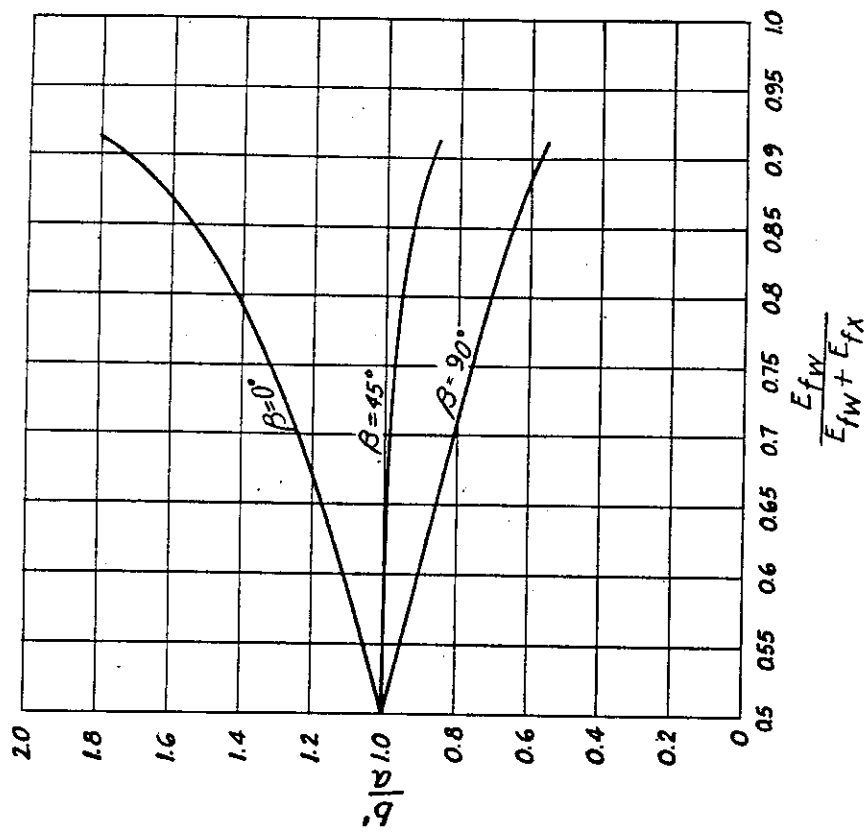


Figure 2-41. Buckling of infinitely long plates of symmetrical construction under uniform compression for determination of b'/a . Edges simply supported.

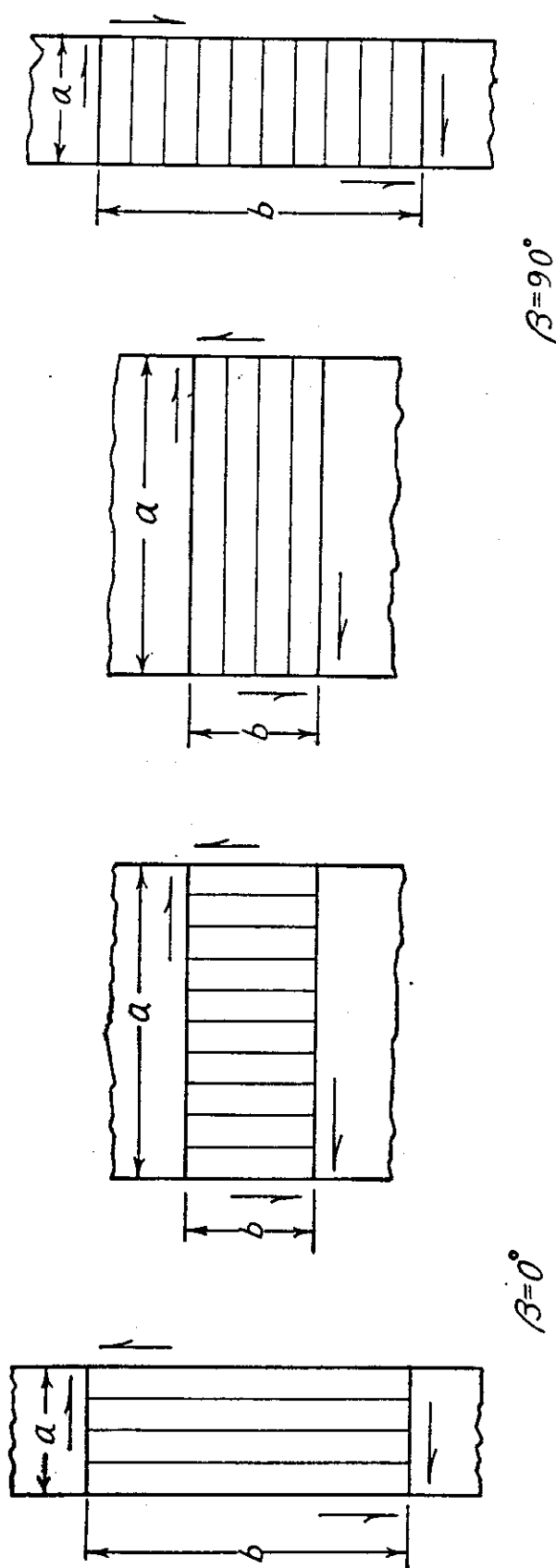


Figure 2-42. In panels loaded in shear, a may be a dimension of either edge. For $\beta = 0^\circ$, face grain is perpendicular to a ; for $\beta = 90^\circ$, face grain is parallel to a .

Table 2-14. Buckling constants for plywood ¹

THREE-PLY													
Face grain angle	Shear								Compression				
	0°		90°		45°				0°	0° and 90°	90°	45°	
					Face grain in tension		Face grain in compression						
Nominal thickness	(K _∞) _∞	b'/a	(K _∞) _∞	b'/a	(K _∞) _∞	b'/a	(K _∞) _∞	b'/a	b'/a	(K _c) _∞	b'/a	(K _c) _∞	b'/a
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
<i>Inch</i>													
0.035	0.60	2.13	2.05	0.60	0.57	0.95	3.50	1.74	1.88	0.71	0.53	0.86	0.84
.070	.80	1.79	2.11	.68	.78	1.06	3.34	1.72	1.62	.82	.62	1.08	.91
.100	.75	1.85	2.10	.66	.73	1.03	3.38	1.72	1.67	.80	.60	1.03	.90
.125	.88	1.70	2.13	.71	.87	1.10	3.27	1.71	1.55	.87	.65	1.17	.93
.155	.94	1.65	2.14	.72	.93	1.12	3.22	1.70	1.50	.89	.67	1.23	.94
.185	.95	1.64	2.14	.73	.94	1.13	3.22	1.70	1.50	.90	.68	1.24	.94

FIVE-PLY													
0.160	1.25	1.42	2.13	0.83	1.29	1.26	2.91	1.66	1.31	1.02	0.77	1.49	0.97
.190	1.35	1.36	2.12	.87	1.41	1.30	2.81	1.64	1.26	1.04	.80	1.56	.98
.225	1.37	1.35	2.11	.88	1.43	1.31	2.79	1.63	1.25	1.05	.81	1.57	.98
.250	1.30	1.38	2.12	.85	1.35	1.28	2.86	1.64	1.28	1.04	.79	1.53	.98
.315	1.29	1.39	2.12	.85	1.34	1.28	2.87	1.65	1.29	1.03	.78	1.52	.98
.375	1.48	1.28	2.08	.92	1.57	1.36	2.66	1.60	1.19	1.08	.84	1.64	.99

SEVEN-PLY (all plies of equal thickness)													
Any	1.40	1.32	2.10	0.89	1.46	1.32	2.75	1.62	1.23	1.06	0.82	1.59	0.99

NINE-PLY (all plies of equal thickness)													
Any	1.52	1.26	2.06	0.94	1.63	1.37	2.60	1.59	1.17	1.09	0.86	1.66	0.99

ELEVEN-PLY (all plies of equal thickness)													
Any	1.59	1.22	2.03	0.96	1.72	1.40	2.52	1.58	1.14	1.10	0.88	1.70	0.99

¹ The buckling constants listed in this table correspond only to the plywood thicknesses and constructions listed in table 2-13 that correspond to Army-Navy specification AN-NN-P-511b (Plywood and Veneer; Aircraft Flat Panel). The values in this table were computed as follows: For each construction given in table 2-13 a value of $\frac{E_{fw}}{E_{fw} + E_{fx}}$ was computed from columns 5 and 6 of table 2-13. These values for each thickness were averaged and the average values were used in entering figures 2-38, 2-39, 2-40, and 2-41 from which the values of this table were obtained. For a more exact determination of these buckling constants or to determine the buckling constants of a plywood construction different from those specified in AN-NN-P-511b, see section 2.7.

The curve marked b'/a is the ratio of half the wave length (b') of a buckle in an infinitely long panel to its width (a). This ratio is to be used in conjunction with figures 2-37 to 2-41 in obtaining

the correction factors for panels of finite length to be applied to K_{∞} .

The curves in figures 2-30 to 2-36 marked γ give the slope of the panel wrinkles with respect

to the $O-X$ axis indicated on the stress sketches.

The procedure in the use of these figures is as follows:

- (1) From the analysis the shear stress (f_s) and the compression (f_c) or tension stress ($-f_c$) acting on a particular plywood panel will have been calculated.
- (2) Determine the ratio f_s/f_c and, on the figure giving the same plywood construction and angle β , draw a line through the origin having a slope (positive or negative) equal to this ratio. When the plywood construction is not the same as that given in the figures, this procedure for determining the buckling constant will have to be run through on the two most similar constructions and an interpolation of the results made on the basis of

$$\frac{E_{f_{ys}}}{E_{f_{yc}} + E_{f_{yt}}}$$

- (3) The point at which the constructed line crosses the curve gives the critical buckling constants $K_{s\infty}$ and $K_{c\infty}$ at which an infinitely long panel will just buckle when subjected to the same ratio of shear to compression that exists on the panel in question.
- (4) Read the value of b'/a for the point on the b'/a curve which is obtained by projecting horizontally from $K_{s\infty}$ determined in step (3).
- (5) From the panel dimensions compute b' and b/b' .
- (6) Figures 2-37 to 2-41 will give the ratio of $K_s/K_{s\infty}$ from which the value of K_s can be computed (K_s is always taken as positive).
- (7) The critical buckling shear stress (F_{scr}) may then be determined by equation (2:80). This represents the maximum allowable shear stress which the panel in question can sustain without buckling when subjected simultaneously to a compressive stress equal to that given in step (1).

2.72. STRENGTH AFTER BUCKLING.

2.721. *General.* Plywood panels may sustain greater loads than those sufficient to cause buckling. When buckling takes place the stresses within the panel are redistributed, the maximum stresses occurring at the edges. The panel will continue to accept load until these stresses reach

the ultimate value. The load at failure is obtained from empirical curves in which the ratio of the average stress at failure to the ultimate strength of the plywood is plotted against the ratio of the width of the panel to the width of a hypothetical panel that will fail at its buckling load.

2.722. *Compression* ($\beta = \text{any angle}$). The abscissa of figure 2-43 is obtained from the equation

$$\frac{a}{a_0} = \sqrt{\frac{F_{cu\theta}}{F_{scr}}} \quad (2:81)$$

in which $F_{cu\theta}$ is obtained from equation (2:51) or (2:52) and F_{scr} from equation (2:77) or (2:79). The ordinates give the ratio of the average stress at which failure will occur to the ultimate compressive strength ($F_{cu\theta}$) of the plywood.

2.723. *Shear* ($\beta = 0^\circ, 45^\circ, \text{ or } 90^\circ$). The abscissa of figure 2-44 is obtained from the equation

$$\frac{a}{a_0} = \sqrt{\frac{F_s}{F_{scr}}} \quad (2:82)$$

in which F_s is obtained from equation (2:50) (2:57), or (2:58) and F_{scr} from equation (2:77) or (2:80). The ordinates give the ratio of the average stress at which failure will take place to the ultimate shear stress (F_s) of the plywood.

2.73. ALLOWABLE SHEAR IN PLYWOOD WEBS.

2.730. *General.* Beams are required to have a high strength-weight ratio and, therefore, they are generally designed so that they will fail in shear at about the load which will cause bending failures. A higher strength-weight ratio is usually obtained if the beams fail in bending before shear failure can occur.

Plywood when used as webs of beams is subjected to different stress conditions from those when it is used in simple shear frames. It is essential, therefore, that tests to determine the strengths of shear webs be made upon specimen beams designed with flanges only sufficiently strong to hold the load at which shear failure is expected. Plywood webs tested in heavy shear frames with hinged corners will give shear strengths that are too high for direct application to beam design.

In any case where buckling is obtained, the stiffeners must have adequate strength to resist the additional loads due to such buckling, and the webs must be fastened to the flanges in such a manner as to overcome the tendency of the buckles in the web to project themselves into this fastening and cause premature failure (ref. 2-23 and 2-42).

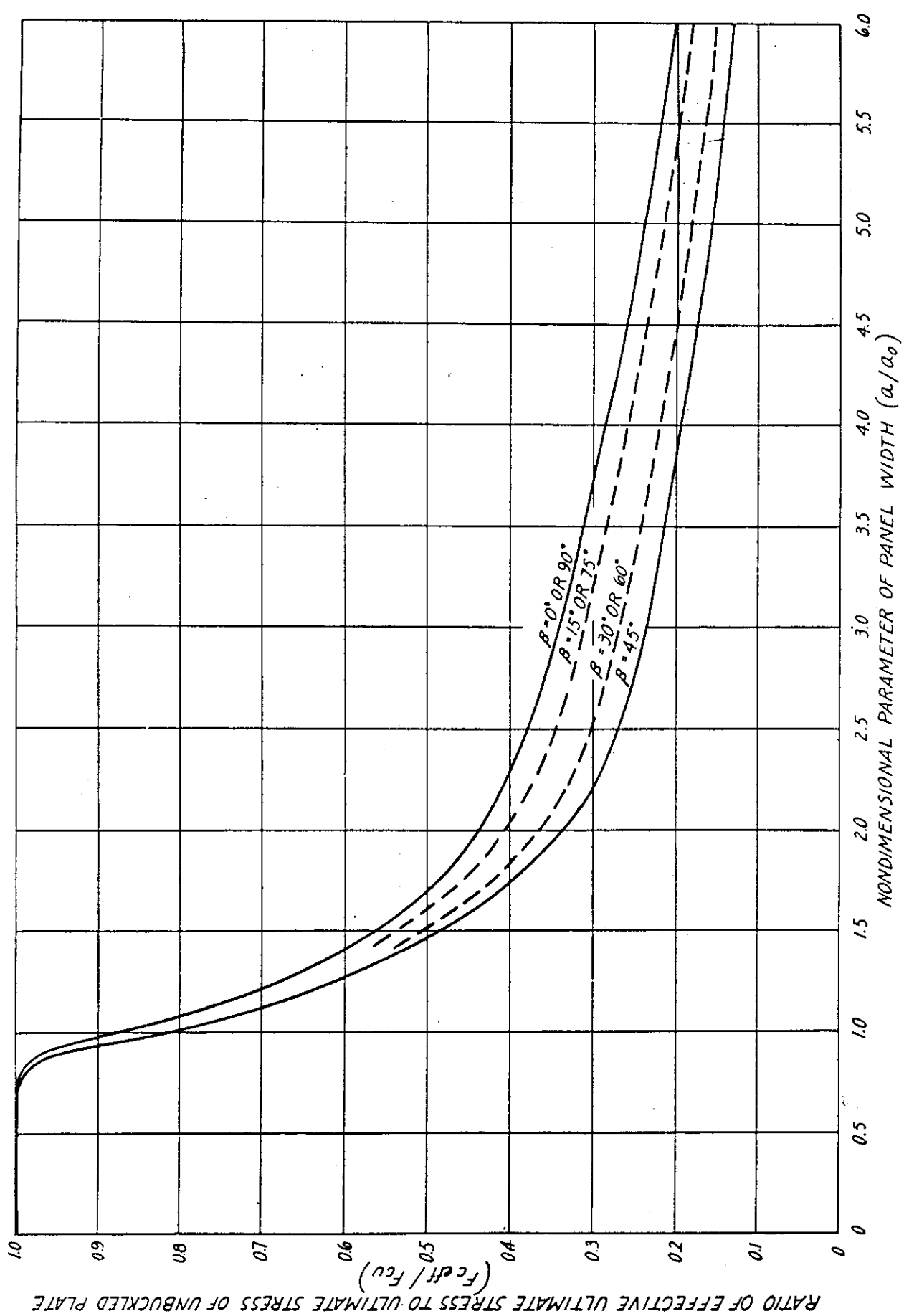


Figure 2-43. Effective width of flat plywood plates in compression at 0° , 15° , 30° , 45° , 60° , 75° , or 90° to direction of face grain. (Curves based on buckling tests of plates and compression tests of coupons 1 inch wide by 4 inches long for 0° or 90° and 6 inches wide by 2 inches long for 15° , 30° , 45° , 60° , or 75° .)

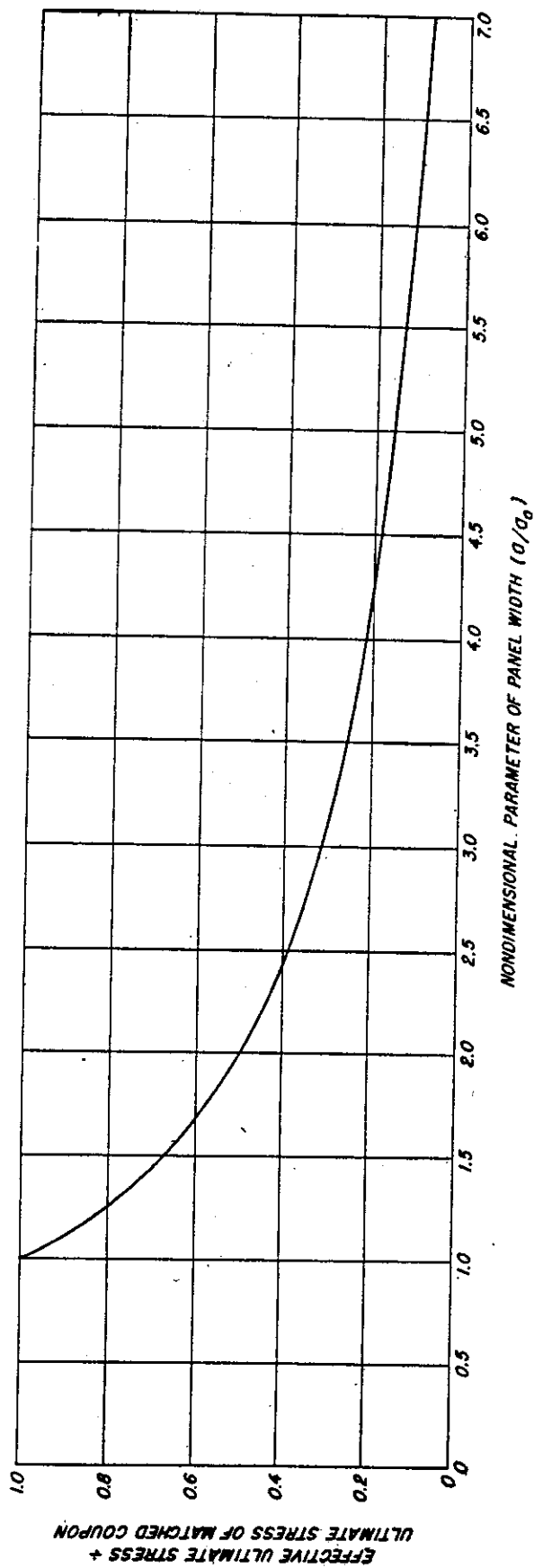


Figure 2-44. Effective ultimate stress divided by ultimate stress of matched coupon.

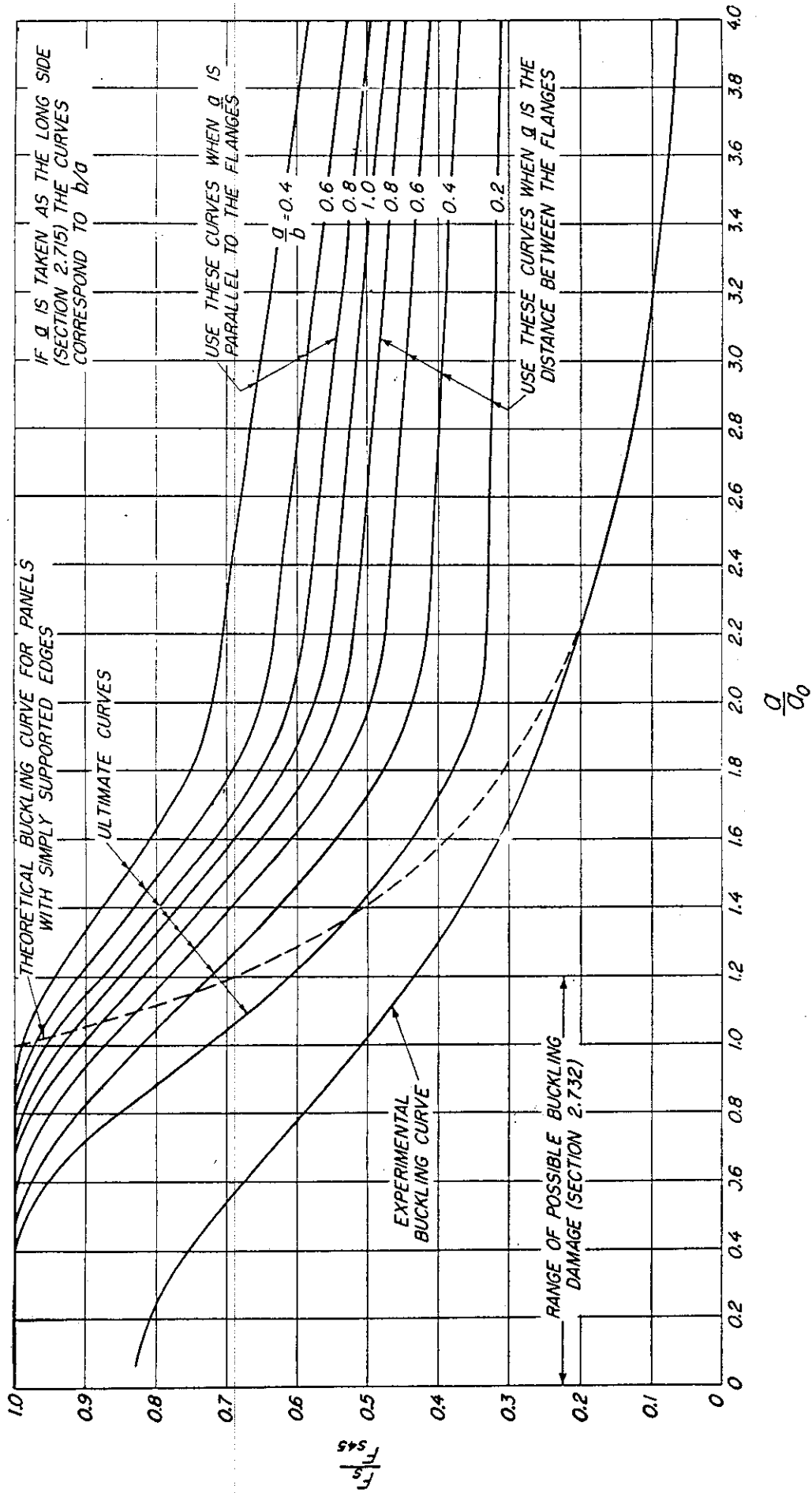


Figure 2-45. Allowable ultimate stress and probable buckling stress for plywood webs in shear. These curves were obtained by using constants for panels with simply supported edges; they do not apply when clamped-edge constants are used.

2.731. *Allowable shear stresses.* The allowable shear stresses of plywood webs having the face grain direction at 0°, 45°, or 90° to the main beam axis may be obtained from figure 2-45.

Values of the parameter $\frac{a}{a_0}$ are obtained from equation (2:82) as explained in section 2.723. Each curve of the family shown in the figure is similar to the curve of figure 2-44. The allowable shear stress (F_s) for the web can be obtained in terms of F_s/F_{s0} from figure 2-45. (For a/a_0 values greater than 4.0, the a/b curves may be extrapolated as straight lines to meet at a point corresponding to $a/a_0=10$ and $F_s/F_{s0}=0.2$.)

The direct use of figure 2-45 for any type of beam having 45° shear webs has been verified by numerous tests of I- and box-beams. A few exploratory tests of beams having 0° and 90° plywood shear webs has indicated that the allowable ultimate shear stresses obtained for these constructions by using figure 2-45 are conservative.

Plywood shear webs of 45° are more efficient than 0° or 90° webs.

The designer is cautioned that box beams may fail at a load lower than that indicated by the strength of the webs as shown in figure 2-45, because of inadequate glue areas of webs at stiffeners or flanges. Such premature failures result from a separation of the web from the flanges or stiffeners.

Figure 2-45 contains a parameter a/b in the form of a family of curves. The $a/b=1$ curve represents a spacing between stiffeners just equal to the clear depth between flanges. The curves below $a/b=1$ should be used for the design of shear webs of beams whose stiffener spacing exceeds the clear distance between flanges. The upper set of curves should be used for the design of beams whose stiffener spacing is less than the clear distance between flanges.

2.732. *Buckling of plywood shear webs.* In connection with shear web tests on various types of beams, it was observed that for plywood webs in the a/a_0 range of less than 1.2, buckling was of the inelastic type that often caused visible damage soon after buckling and sometimes just as the buckles appeared for those webs designed to fail in the neighborhood of F_{s0} . No accurate criteria can be presented at this time, but the designer is cautioned to avoid the use of webs that may be damaged by buckling before the limit or yield stress is reached. The buckling curve established by these tests is shown in figure 2-45.

2.74. *LIGHTENING HOLES.* When the computed

shear stress for a full depth web of practical design is relatively low, as in some rib designs, the efficiency, or strength-weight ratio, may be increased by the careful use of lightening holes and reinforcements. General theoretical or empirical methods for determining the strength of plywood webs with lightening holes are not available, and tests should, therefore, be made for specific cases (ref. 2-64).

2.75. *TORSIONAL STRENGTH AND RIGIDITY OF BOX SPARS.* The maximum shear stresses in plywood webs for most types of box spars subjected to torsion may be calculated from the following formula:

$$f_s = \frac{T}{b't(C' - 2b')} \quad (2:83)$$

where

t =thickness of one web

b' =mean width of spar (total width minus thickness of one web)

C' =average of the outside and inside periphery of the cross section.

The allowable ultimate stress in torsion of plywood webs is determined as in section 2.723.

The torsional rigidity of box beams up to the proportional limit, or to the buckling stress (whichever is the lesser) is given by the formula:

$$\theta = \frac{TC'L}{4Gtb'(C' - 2b')^2} \quad (2:84)$$

2.76. *PLYWOOD PANELS UNDER NORMAL LOADS.*

2.760. *General.* When rectangular plywood panels, which have the face grain direction parallel or perpendicular to the edges, are subjected to normal loads, the deflections and in some cases the stresses developed, are given by the following approximate formulas. If the maximum panel deflection exceeds about one-half its thickness, the formulas for large deflections will give results which are somewhat more accurate than those given by the formulas for small deflections (ref. 2-51).

2.761. *Small deflections.*

(a) Uniform load—all edges simply supported.

$$w_0 = 0.155 K_1 \frac{pa^4}{E_1 t^3} \quad (2:85)$$

where

w_0 =deflection at center of panel

p =load per unit area

a =width of plate (short side)

K_1 =constant from figure 2-46 (a)

The maximum bending moment at the center of the panel on a section perpendicular to side a

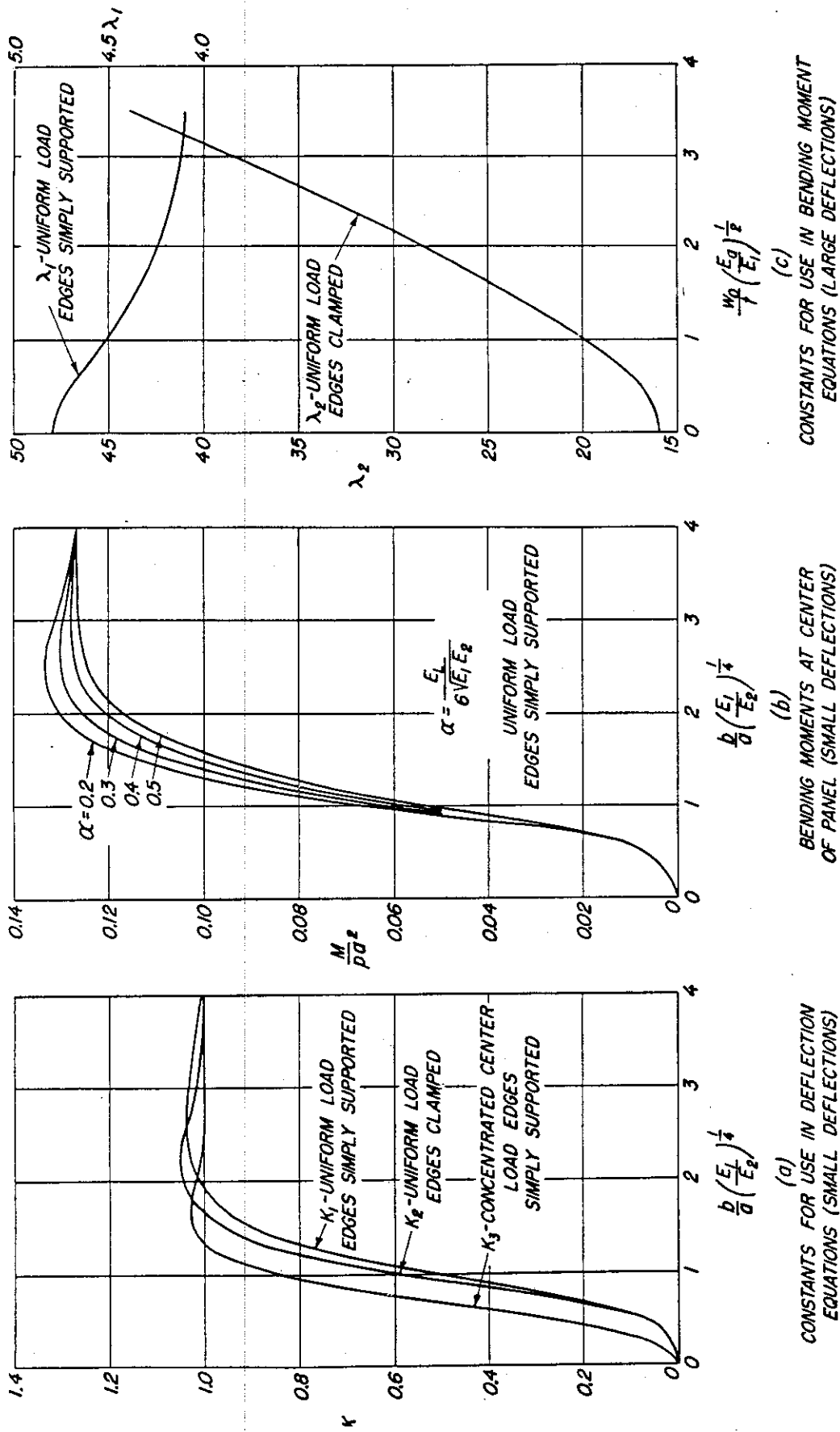


Figure 2-46. Curves of bending moments and deflection constants for flat rectangular plywood panels subjected to normal loads.

may be obtained from figure 2-46 (b). The maximum bending moment on a section perpendicular to side b is given by the same curve, provided a and b , and E_1 and E_2 are interchanged in the abscissa, and a is replaced by b in the ordinate. The corresponding stresses can be calculated from the formulas given in section 2.614.

(b) Uniform load—all edges clamped.

$$w_o=0.031\,K_2\frac{pa^4}{E_1t^3}\tag{2:86}$$

where

K_2 =constant from figure 2-46 (a)

(c) Concentrated load at center—all edges simply supported.

$$w_o=0.252K_3\left(\frac{E_1}{E_2}\right)^{1/4}\frac{Pa^2}{E_1t^3}\tag{2:87}$$

where

K_3 =constant from figure 2-46 (a).

2.762. *Large deflections.*

(a) Uniform load—all edges simply supported.

The relation between the load and deflection is given by the formula:

$$p=K_4E_Lw_o\frac{t^3}{a^4}+K_5E_Lw_o^3\frac{t}{a^4}\tag{2:88}$$

where

K_4 and K_5 are constants whose approximate values are given in table 2-15.

E_L is taken for the species of the face ply.

The maximum bending moment at the center of the panel can be calculated from the following approximate formula provided the length of the panel exceeds its width by a moderate amount.

$$M_{max.}=\lambda_1E_1w_o\frac{t^3}{6a^2}\text{ (long narrow panels only)}\tag{2:89}$$

where

λ_1 =constant from figure 2-46 (c).

Although the edge support conditions are taken as simply supported, it is assumed that the panel length and width remain unchanged after the panel has been deflected. Therefore, in addition to the bending stress, there will be a direct tensile or membrane stress set up in the plane of the plywood, and the total stress in any ply will be the algebraic sum of the bending stress and direct stress in that ply. The maximum total stress will occur in the extreme fiber of the

outermost ply having its grain direction perpendicular to the plane of the section upon which the moment was taken; the bending stress being calculated from formula (2:67) and the direct stress from section 2.601 after first determining the average direct stress across the section from the formula:

$$f_{t(av.)}=2.55\,E_a\left(\frac{w_o}{a}\right)^2\text{ (long narrow panels only)}\tag{2:90}$$

(b) Uniform load—all edges clamped.

The load-deflection relation, formula (2:88), will also apply to this case provided K_6 and K_7 from table 2-15 are substituted for K_4 and K_5 , respectively. The maximum total stress may also be determined as outlined in (a) above, provided λ_2 from figure 2-46 (c) is substituted for λ_1 in formula (2:89).

Table 2-15. Values of constants in the approximate deflection formulas for plywood panels under normal loads ¹

Panel construction ²	Uniform load all edges simply supported			Uniform load all edges clamped		
	(b/a)	K_4	K_5	(b/a)	K_6	K_7
3 ply, $\theta=0^\circ$	1.0	(see $\theta=90^\circ$)	(see $\theta=90^\circ$)	1.0	(see $\theta=90^\circ$)	(see $\theta=90^\circ$)
	1.5	1.7	5.9	2.0	3.6	6.0
	2.0	.9	4.7	>3.0	2.5	7.0
	>3.0	.5	4.7			
	$\theta=90^\circ$	>1.0	6.3	13.3	1.0	33.3
5 ply, $\theta=0^\circ$	1.0	(see $\theta=90^\circ$)	(see $\theta=90^\circ$)	1.0	(see $\theta=90^\circ$)	(see $\theta=90^\circ$)
	1.5	2.4	6.5	2.0	8.3	8.2
	>2.0	1.5	6.0	>3.0	7.9	9.4
	$\theta=90^\circ$	1.0	6.2	12.3	1.0	28.7
	>1.5	5.0	10.0	>2.0	26.5	15.5

¹ The values given in this table are for spruce plywood, all plies of equal thickness, but they may also be considered applicable to plywood of other species and of the same constructions. For plywood made of more than five plies or of unequal ply thickness, the above table may be used as a rough guide in arbitrarily selecting values of these constants.

² θ is the angle between the face grain direction and side b of the panel.

2.77. STIFFENED FLAT PLYWOOD PANELS.

2.771. *The stiffness of a stiffener affixed to a plywood panel (ref. 2.71).* When a stiffener is affixed to a panel the neutral surface of the panel moves toward the stiffener as illustrated in figure 2-47. The amount of this movement is given by the equation:

$$Z_n=\frac{1}{2}\frac{t+d}{2at^4\sqrt{E_a^3E_b}+\frac{E_at}{c\pi hE_s\alpha d}+1+\frac{E_at}{E_bd}}\tag{2:91}$$

in which

$$\alpha = \sqrt{\psi^2 + \sqrt{\psi^2 - 1}}$$

$$\psi = \frac{1}{2} \sqrt{E_w E_z} \left(\frac{1}{G_{xz}} - \frac{2\mu_{xz}}{E_z} \right)$$

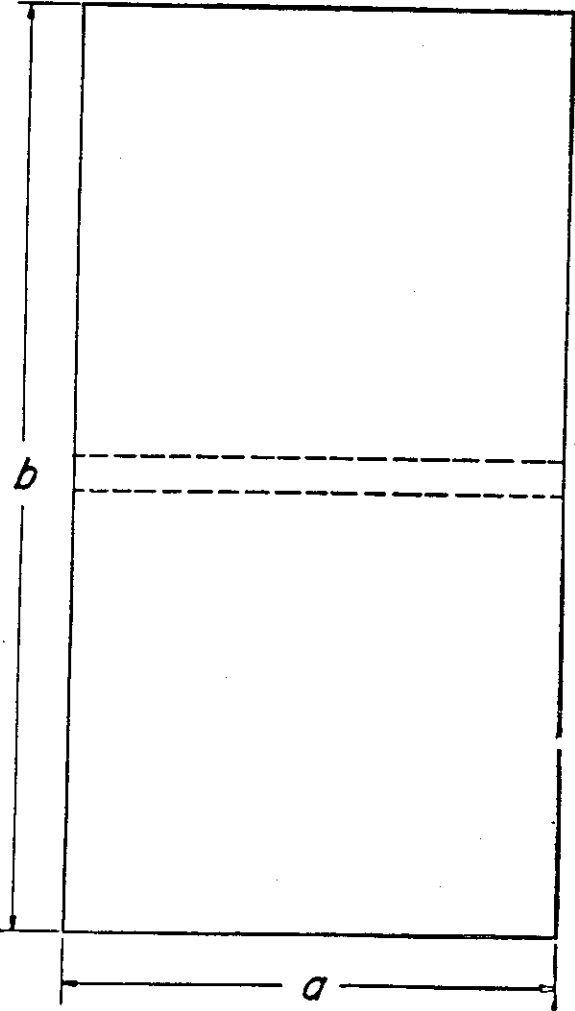
$c=1$ if the edge of length b is simply supported

$c=2$ if the edge of length b is clamped

E_s =modulus of elasticity of the stiffener in the direction of its length

The stiffness in the neighborhood of the stiffener added to the plate by the presence of the stiffener is approximately:

$$(EI)_s = \frac{hdE_s}{12} [d^2 + 3(t+d-2Z_n)^2] + thE_sZ_n^2 \quad (2:92)$$



2.772. A single stiffener bisecting a panel. If the stiffener is sufficiently stiff it will substantially divide the panel into two identical panels that can be designed according to the methods of sections 2.71 and 2.72. The minimum stiffness of the stiffener that will accomplish this purpose is defined in the following sections.

2.7721. Stiffened panel subjected to edgewise compression, stiffener perpendicular to the direction of the stress and parallel to side a ($\beta=0^\circ$ or 90°) (ref. 2-30).

$$(EI)_{scr} = \frac{t a^4}{11b} (F_{crm} - F_{crp}) \quad (2:93)$$

in which $(EI)_{scr}$ is the critical value of $(EI)_s$ as determined by equation (2:92); F_{crp} and F_{crm} are the critical buckling stresses of the panel, considered to be unstiffened and adequately stiffened

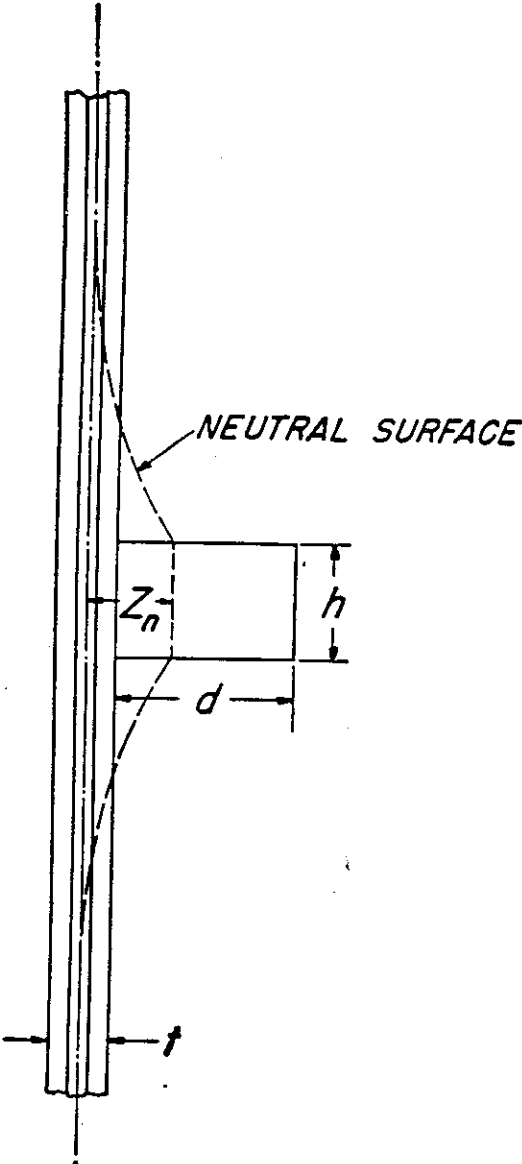


Figure 2-47. Nomenclature for equations (2:91) and (2:92).

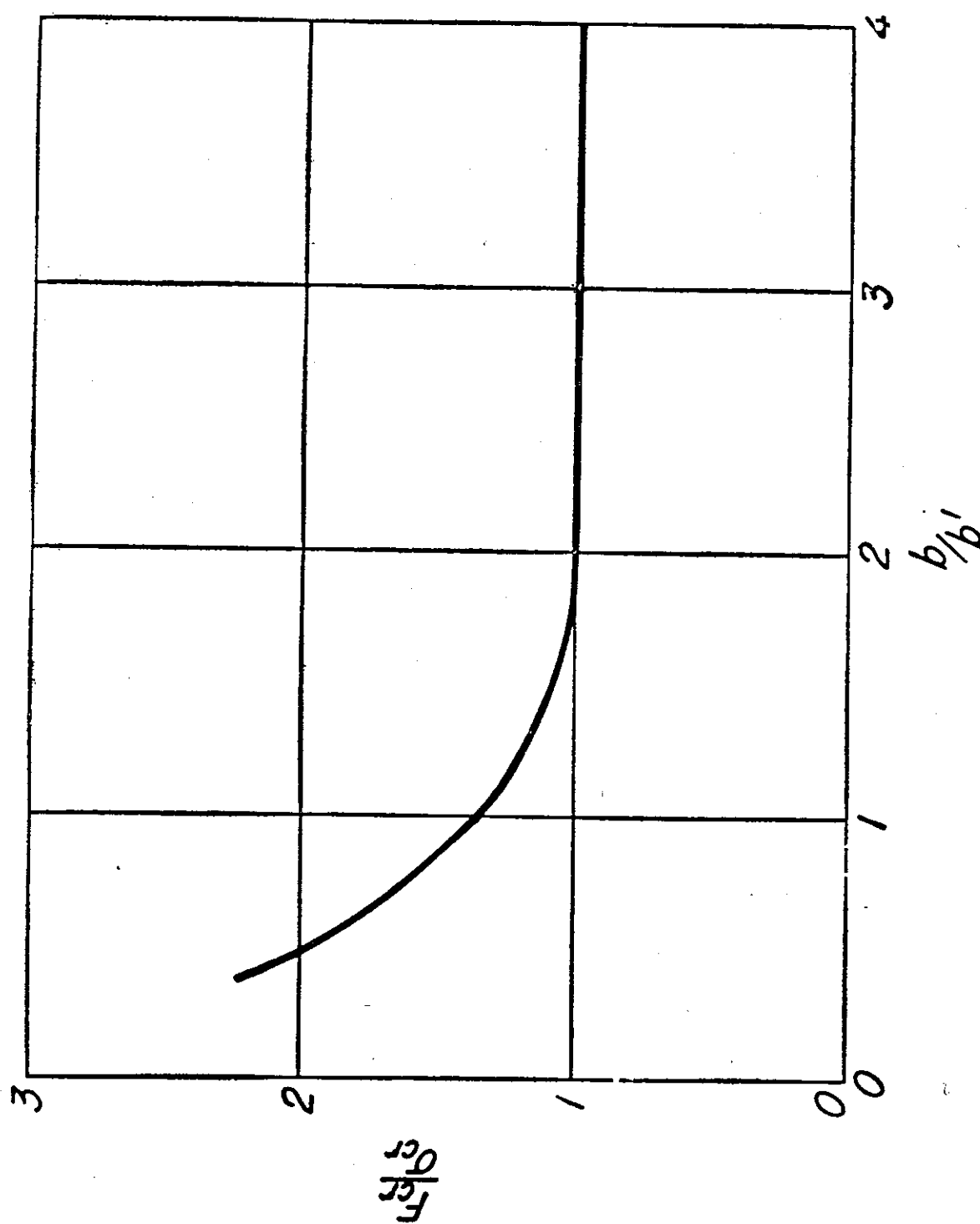


Figure 2-48. Curve for the determination of values of σ_{cr} and σ_{erm} from values of F_{cr} and F_{erm} .

respectively, obtained by means of the method of section 2.712, Case I.

2.7722. *Stiffened panel subjected to edgewise compression, stiffener perpendicular to the direction of the stress ($\beta=45^\circ$).* The stiffness in the neighborhood of the stiffener added to the plate by the stiffener is assumed to be the stiffness of the stiffener alone

$$(EI)_s = \frac{1}{12} E_s h d^3 \quad (2:94)$$

and the critical stiffness of the stiffener is approximately

$$(EI)_{scr} = \frac{t a^4}{40 b} (\sigma_{crm} - \sigma_{crp}) \quad (2:95)$$

in which

a = the dimension of the panel perpendicular to the direction of the stress

b = the dimension of the panel parallel to the direction of the stress

and in which the values of σ_{crp} and σ_{crm} are obtained from the critical buckling stresses of the panel, considered to be unstiffened and adequately stiffened respectively, by means of figure 2-48 and the method of section 2.715 for panels with edges simply supported and having their face grain at 45° to their edges.

2.7723. *Stiffened panel subjected to edgewise compression, stiffener parallel to the direction of the stress and to side b ($\beta=0^\circ$ or 90°).*

$$(EI)_{scr} = \frac{t a b^2}{10(n+1)^2} \left[F_{crm} \left(1 + \frac{2 d h E_s}{a t E_b} \right) - F_{crp} \right] \quad (2:96)$$

in which $(EI)_{scr}$ is the critical value of $(EI)_s$ as determined by equation (2:92); F_{crp} and F_{crm} are the critical buckling stresses of the panel, considered to be unstiffened and adequately stiffened respectively, obtained by the means of the method of section 2.712, Case I, and n is the number of half-waves that occur in the unstiffened panel. It may be noted that the dimensions of the stiffener, d and h , appear in equation (2:96) as well as in equations (2:91) and (2:92) and, therefore, it is necessary to estimate the values of these dimensions and verify the estimate by use of equation (2:96).

The compressive stress in the stiffener associated with the critical stress of the panel is:

$$f_{scr} = \frac{E_s}{E_b} F_{crm} \quad (2:97)$$

The load carried by the panel and the stiffener at the critical stress of the panel is:

$$P_{cr} = F_{crm} \left[a t + \frac{E_s}{E_b} d h \right] \quad (2:98)$$

The ultimate load of the stiffened panel cannot be greater than the sum of the ultimate loads of the two half panels according to section 2.722 plus the ultimate load of the stiffener considered as a short column. The reduction of this sum in terms of the a/a_o of one of the half panels is given by figure 2-49 (ref. 2-30).

2.7724. *Stiffened panel subjected to edgewise compression, stiffener parallel to the direction of the stress ($\beta=45^\circ$).* The stiffness in the neighborhood of the stiffener added to the panel by the stiffener is assumed to be the stiffness of the stiffener alone and is given by formula (2:94). The critical stiffness of the stiffener is:

$$(EI)_{scr} = \frac{1}{66} t a b^2 (F_{crm} - F_{crp}) \quad (2:99)$$

in which $(EI)_{scr}$ is the critical value of $(EI)_s$ as determined by equation (2:94); F_{crm} and F_{crp} are the critical buckling stresses of the panel, considered to be adequately stiffened and unstiffened, respectively, obtained by means of the method of section 2.715; and a and b are the dimensions of the panel perpendicular and parallel to the direction of the stress, respectively (ref. 2-70).

2.7725. *Stiffened panel subjected to edgewise shear. Stiffener parallel to edges (or ends) of panel and $\beta=0^\circ$ or 90° .*

$$(EI)_{scr} = 8 \frac{E_s h t^3 L^4}{K_1 a^4} \quad (2:100)$$

in which

$(EI)_{scr}$ = the critical value of $(EI)_s$ as determined by equation (2:92)

L = length of stiffener

a = width of panel (independent of the direction of the stiffener)

K_1 is determined by means of figure 2-46

The critical stress of the stiffened panel is computed by means of section 2.713 equation (2:77) applied to one half of the panel as divided by the stiffener, provided that $(EI)_s$ is equal to or greater than $(EI)_{scr}$.

2.773. *A plywood panel stiffened with a multiplicity of closely spaced stiffeners parallel to one of its edges ($\beta=0^\circ$ or 90°).* If the spacing of the stiffeners is not too great the formulas for plywood

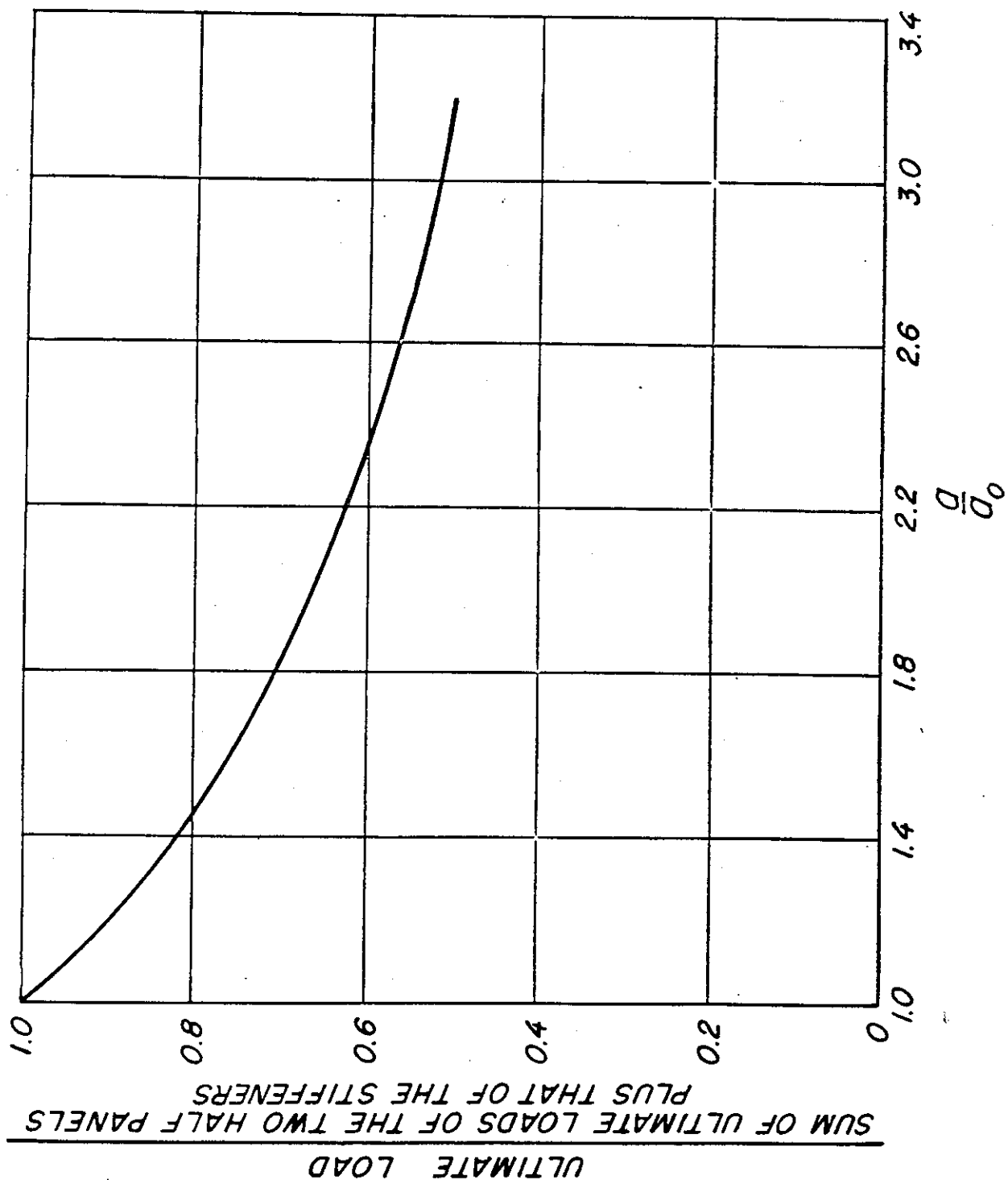


Figure 2-49. Curve for computation of ultimate load on a panel having a vertical stiffener.

of section 2.71 can be applied to such panels. It is convenient to employ formulas for the load per inch of length of the edge of the stiffened panel rather than formulas for stress, thus

$$P_{scr} = F_{scr} t_e$$

where t_e is the effective thickness of the stiffened panel, which it will not be necessary to compute. The following nomenclature is convenient.

$$D_w = \frac{E_{fw}t^3}{12\lambda_f} \text{ or } \frac{E_{fx}t^3}{12\lambda_f} \left\{ \begin{array}{l} \text{for stiffeners parallel} \\ \text{or perpendicular} \\ \text{to the direction of} \\ \text{the face grain of} \\ \text{the panel, respec-} \\ \text{tively.} \end{array} \right.$$

$$D_{wx} = \frac{G_{fwt}^3}{12}$$

D_{wc} , D_{xc} , and D_{wxc} are computed similarly to the above except that the stiffener is considered as an extra ply of the plywood. The location of the neutral axis is taken into account as described in section 2.52.

D_{we} , D_{xe} , and D_{wxe} are effective values that apply to the stiffened panel.

The subscripts w and x applied to D denote directions parallel and perpendicular, respectively, to the direction of the stiffeners, and subscripts 1 and 2 denote directions parallel and perpendicular, respectively, to the direction of the stress for panels subjected to compression and to the direction of side b for panels subjected to shear.

Equations (2:72) and (2:77) become

$$P_{scr} = 12H_c \sqrt{D_{we}D_{xe}} \frac{1}{a^2} \quad (2:101)$$

$$P_{scr} = 12H_s [D_{1e}^3 D_{2e}]^{1/4} \frac{1}{a^2} \quad (2:102)$$

in which

$$k = \frac{D_{we} \mu_{fxwe} + 2D_{wxe}}{\sqrt{D_w D_x}}$$

$$r = \frac{b}{a} \left(\frac{D_{1e}}{D_{2e}} \right)^{1/4}$$

2.7731. *Determination of D_{we} .* The stiffeners being closely spaced, the usual engineering formula that takes into account the location of the neutral axis, can be employed.

$$D_{we} = D_w + \frac{nhd}{12g} E_s \left[d^2 + \frac{3(t+d)^2}{\frac{nhdE_s}{gE_s t} + 1} \right] \quad (2:103)$$

in which

g = the width of the panel across the stiffeners and is equal to a or b as required

n = the number of stiffeners

E_s = modulus of elasticity of the stiffeners in the direction of their length.

2.7732. *Determination of D_{xe} .* When a panel is bent across the stiffeners, the variation of the stiffness at the stiffeners and between the stiffeners, and the presence of a sharp kink at the edges of the stiffeners due to stress concentrations, are taken into account.

$$\frac{D_x}{D_{xe}} = \frac{6}{19} \left[1 - \left(1 - \frac{h}{s} \right)^2 \right] \left[1 - \frac{D_x^2}{D_{xc}^2} \right] - \frac{h}{s} \left[1 - \frac{D_x}{D_{xc}} + 1 \right] \quad (2:104)$$

in which

s = distance center to center of two adjacent stiffeners.

2.7733. *Determination of D_{wxe} .*

$$D_{wxe} = \frac{1}{g} \{ [g - nh] D_{wx} + n [h - \epsilon(t + d)] D_{wxc} \} \quad (2:105)$$

in which g is the width of the panel across the stiffeners, n is the number of stiffeners, and ϵ is determined from figure 2-50 and 2-51.

2.7734. *Determination of μ_{fxwe} .*

$$\mu_{fxwe} = \frac{\sqrt{\mu_{fxw} \mu_{fxw} D_{we} D_{xe}}}{D_{we}} \quad (2:106)$$

in which the values of μ_{fxw} and μ_{fxw} are taken from equations (2:32) and (2:33) of section 2.52, assuming that the stiffener is an added ply of the plywood.

2.774. *Stiffened plywood panels subjected to bending in the direction of the stiffeners.* The maximum bending stress in stiffened plywood panels can be calculated from the following formula, when the face grain direction is 0° or 90° to the direction of the span:

$$f_b = \frac{ME_L c'}{D_{we}} \quad (2:107)$$

where:

c' = distance from the neutral axis of the composite section to the extreme longitudinal fiber

E_L is taken for the species of the outermost longitudinal fiber.

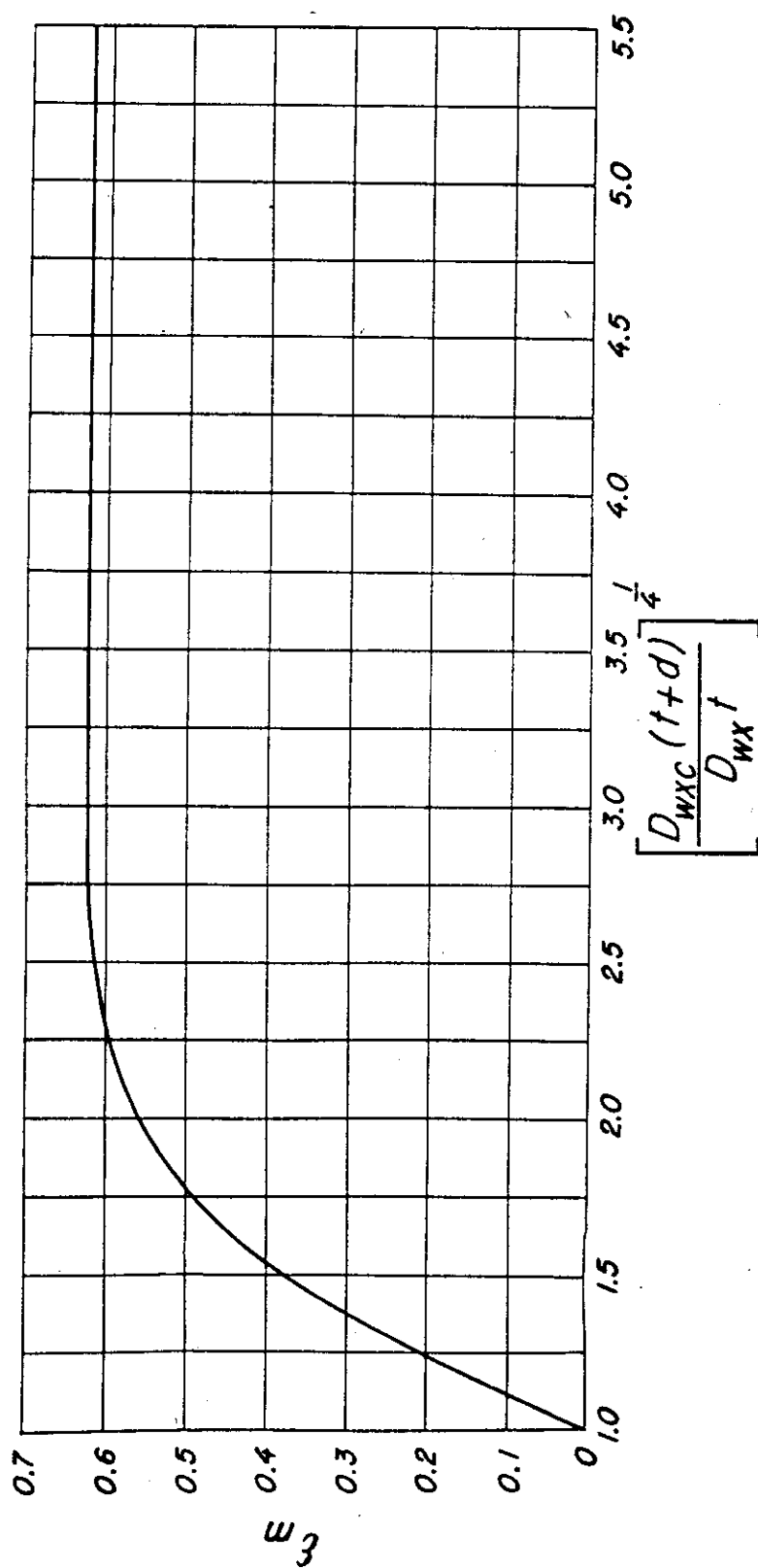


Figure 2-50. Curve for the determination of the values of ϵ_m

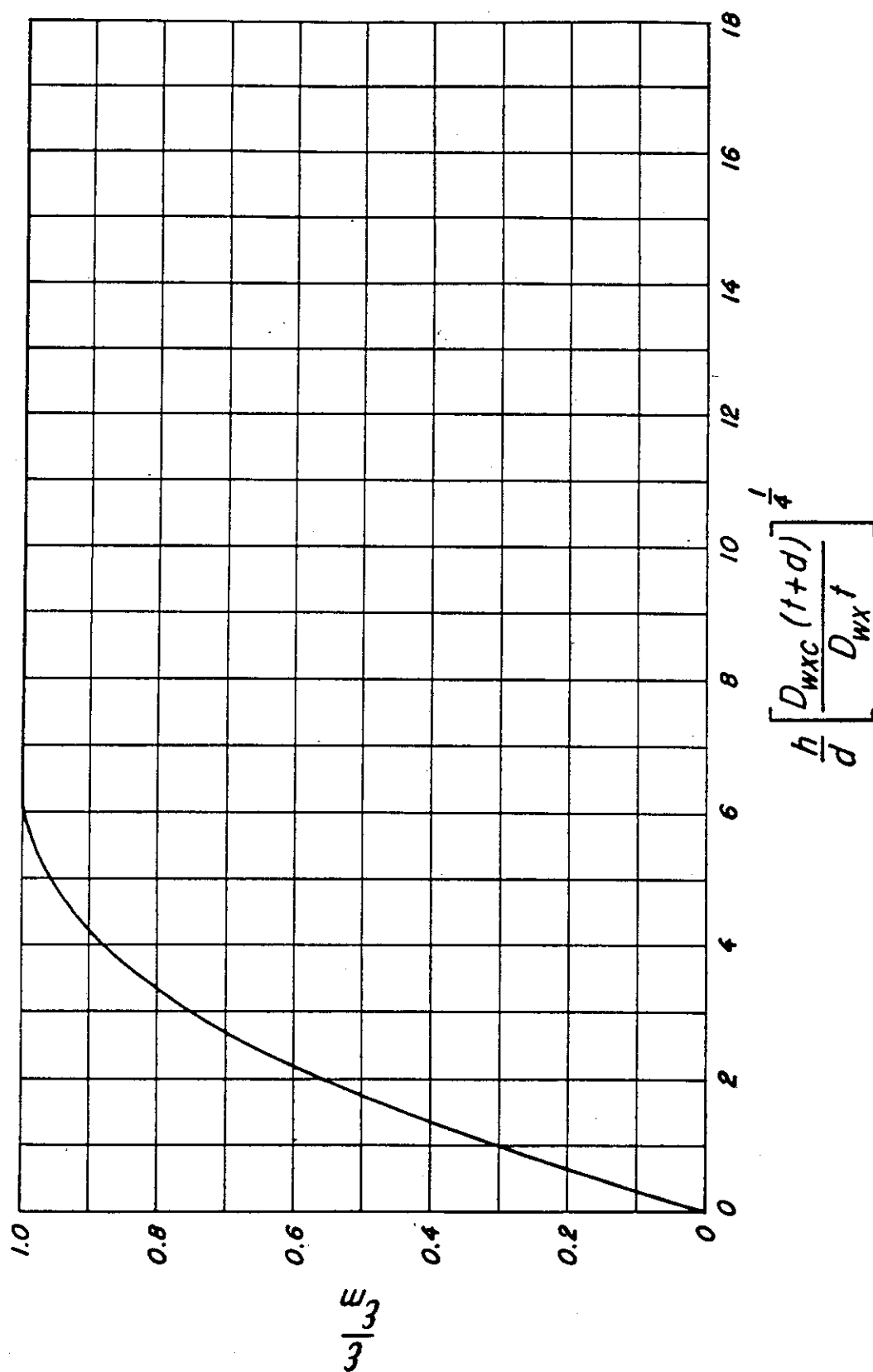


Figure 2-51. Curve for the determination of the values of ϵ/ϵ_m^0

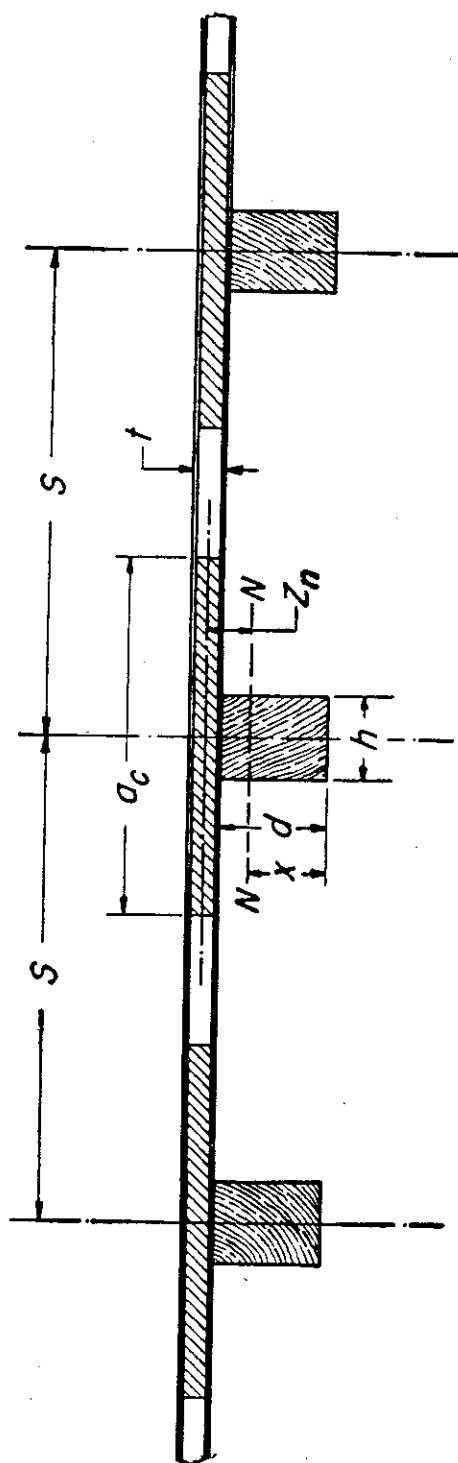


Figure 2-52. Cross section of panel stiffened with multiple stiffeners.

This maximum bending stress should not exceed the modulus of rupture of the material in which the maximum stress exists. If the stiffener is of an I or box section, the modulus of rupture must be corrected by a form factor as follows: When the load is applied so that the outer flange of the stiffener will fail in compression, the proper form factor to use is that for a beam having the same flange dimensions as the outer flange of the stiffener, and the same web thickness as the stiffener, but of a depth equal to $2x$. If the load is applied so that the panel will fail in compression, the proper form factor to use is that of a beam having flange dimensions equal to that of the effective sheet plus the flange of the stiffener adjacent to the panel, and a web thickness equal to that of the stiffener but a depth of $2(d+t-x)$. In either case no form factor need be used if the neutral axis lies within the compression flange, where x is the distance from the neutral axis to the stiffener face away from the panel as shown in figure 2-52.

The effective width of the panel for stresses below the proportional limit is:

$$a_e = \frac{dhE_s(t+d-2Z_n)}{2tE_bZ_n} \quad (2:108)$$

in which Z_n is obtained from equation (2:91) and b is in the direction of the stiffeners.

If the spacing of the stiffener (s in fig. 2-52) is less than a_e , the value of D_{we} is obtained from equation (2:103) and

$$Z_n = \frac{1}{2} \frac{t+d}{\frac{ts}{dh} \frac{E_b}{E_s} + 1} \quad (2:109)$$

If this is not the case, then

$$D_{we} = D_w + \frac{1}{a_e} \frac{hd^3}{12} E_s + tE_bZ_n^2 + \frac{dh}{4a_e} E_s(t+d-2Z_n)^2 \quad (2:110)$$

in which Z_n is obtained from equation (2:91), a_e from equation (2:108), and b is in the direction of the stiffeners.

For stiffened panels having the face grain direction 45° to the length of the stiffeners, the plywood is neglected in the computations and the stiffeners designed to carry the total load.

2.775. *Modes of failure in stiffened panels.* Modes of failure other than failure of the panel or the stiffeners are not considered here.

A possible mode of failure, which has been investigated for only one particular type of construction,

is the premature separation of the plywood panel from its stiffeners occurring when the forces required to restrain the edges of the buckled panels become too great for the strength of the plywood or its attachment to the stiffeners.

Since no criteria suitable for general application are available for predicting the critical modes of failure, it is recommended that typical panels of each particular type of construction be tested.

2.8. Curved Plywood Panels

2.81. *STRENGTH IN COMPRESSION OR SHEAR; OR COMBINED COMPRESSION (OR TENSION) AND SHEAR.* When failure by buckling does not occur, the ultimate strength of curved plywood panels subjected to compression or shear, or combined compression (or tension) and shear may be obtained by the method given in section 2.613. This method is applicable when the face grain direction is at any angle.

2.82. *CIRCULAR THIN-WALLED PLYWOOD CYLINDERS.*

2.821. *Axial compression.*

2.8211. *Compression with face grain parallel or perpendicular to the axis of the cylinder.* The theoretical buckling stress for a long cylinder (to be modified for design as described later in the section) is given by the formula:

$$F_{ccr} = K_\sigma [E_{fw} + E_{fx}] \frac{t}{r} \quad (2:111)$$

where

E_L is for the species of the face plies

t = thickness of plywood

r = radius of cylinder

K_σ is a buckling constant that is a function of

$\frac{E_1}{E_1 + E_2}$ and is determined from figure

2-53. In using figure 2-53, E_1 is the flexural stiffness of the plywood in the direction parallel to the longitudinal axis of the cylinder. E_1 is equal to E_{fw} when the face grain is longitudinal and is equal to E_{fx} when the face grain is circumferential. E_2 is the flexural stiffness of the plywood in the circumferential direction. $E_1 + E_2$ is equal to $E_{fw} + E_{fx}$.

Because of the steepness of the curve for K_σ at the extreme right and left portions, it appears advisable to avoid, when possible, the use of types of plywood for which the ratio $\frac{E_1}{E_1 + E_2}$ is small or nearly equal to unity.

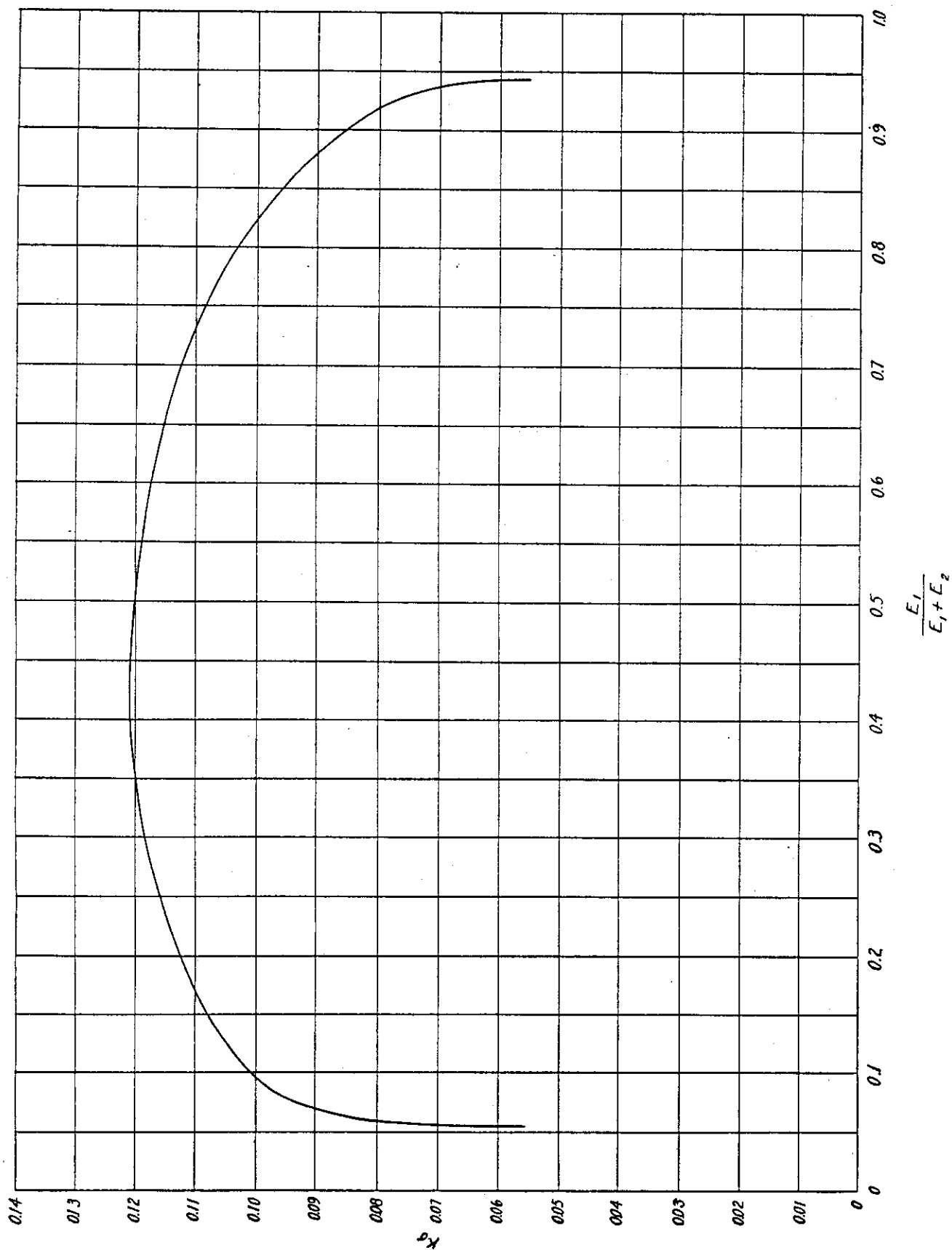


Figure 2-53. Theoretical curve for long, thin plywood cylinders in axial compression.

For use in design, the theoretical buckling stress must be modified as the proportional-limit stress is approached. This is accomplished by the use of figure 2-54. The proportional-limit stress used with this chart is the compressive proportional limit for the plywood in the direction of the cylinder axis and is determined from table 2-13 or from section 2.600. $F_{cp} = F_{cpw}$ when the face grain is longitudinal. $F_{cp} = F_{cpx}$ when the face grain is circumferential. The chart is entered along the abscissa with the ratio $F_{scr}(\text{theoretical})/F_{cp}$. The design buckling stress, F_{scr} , is then obtained by multiplying the ordinate by F_{cp} .

Limited amounts of double curvature have negligible effect on buckling loads.

2.8212. *Compression with 45° face grain.* When the face grain is at an angle of 45° to the cylinder axis, the theoretical buckling stress may be taken as the average of the theoretical buckling stresses obtained by assuming the face grain direction to be: (1) parallel to the cylinder axis, (2) circumferential. In using figure 2-54, however, to obtain the design buckling stress, the proportional-limit value F_{cp} should be that for the plywood at 45° to the face grain. F_{cp45} may be taken as $0.55 F_{cu45}$, where F_{cu45} is determined by section 2.610.

2.8213. *Compression—effect of length.* If the cylinders are not long, an adjusted value by K_r , designated by K_{sa} , should be used in formula (2:111). Values of K_{sa} can be determined from figure 2-55 in which L is the length of the cylinder, and the subscripts 1 and 2 apply to the axial and circumferential directions respectively.

2.822. *Bending.* For bending, the design buckling stress determined as for compression may be increased 10 percent.

2.823. *Torsion.* The buckling stresses of thin plywood cylinders can be computed by the formula

$$F_{scr} = K_r (E_{fw} + E_{fz}) \frac{t}{r} \quad (2:112)$$

in which the value of K_r depends upon values of W , U , $\frac{E_1}{E_1 + E_2}$, and θ

$$W = \frac{1}{2} \frac{G_{fw} + G_{wz}}{E_{fw} + E_{fz}}$$

$$U = \frac{L^2}{rt}$$

θ = angle between face grain and generator of cylinder (fig. 2-56)

Values of K_r for different values of W , U , and θ are given in figures 2-57, 2-58, and 2-59. The nomenclature is illustrated in figure 2-56. (Ref. 2-93)

2.824. *Combined torsion and bending.* Cases of combined loading can be checked by the following interaction formula:

$$\left(\frac{f_{st}}{F_{scr}} \right)^{4/3} + \left(\frac{f_b}{F_{bcr}} \right)^{4/3} = 1.0 \quad (2:113)$$

where:

f_{st} = applied torsional shear stress

f_b = applied bending stress

F_{scr} = pure torsion design buckling stress

F_{bcr} = pure bending design buckling stress

2.83. CURVED PANELS.

2.831. *Axial compression.* The buckling stress is that of a complete cylinder, of which the curved panel can be considered to be a part, of a length equal to the axial dimension of the panel. It can be obtained by use of formula (2:111) corrected for length by the method of section 2.8213.

If the curved panel is very accurately made, higher values may be obtained by test but cannot be counted upon in design.

2.832. *Shear.* An approximation of the buckling stress is obtained by adding the buckling stress of the panel considered to be flat to that of the cylinder of which the panel can be considered to be a part. Thus the buckling stress is given approximately by (ref. 2-40)

$$F_{scr} = H_s \frac{(E_1^3 E_2)^{1/4}}{3\lambda_f} \left(\frac{t}{a} \right)^2 + K_r (E_{fw} + E_{fz}) \frac{t}{r} \quad (2:114)$$

or

$$F_{scr} = [E_{fw} + E_{fz}] \left[K_s \frac{t^2}{a^2} + K_r \frac{t}{r} \right] \quad (2:115)$$

in which formula (2:114) comes from (2:77) and (2:112); formula (2:115) comes from (2:80) and (2:112).

2.84. LONGITUDINALLY STIFFENED CYLINDERS. A multiplicity of evenly spaced identical stiffeners attached to the inner surface of the cylinders.

2.841. *Stresses when buckling does not occur.* The strengths can be computed by use of sections 2.610 to 2.614.

2.8411. *Axial compression (or tension).* The compressive stress in the plywood is:

$$f_c = \frac{PE_a}{nhdE_s + 2\pi rtE_a} \quad (2:116)$$

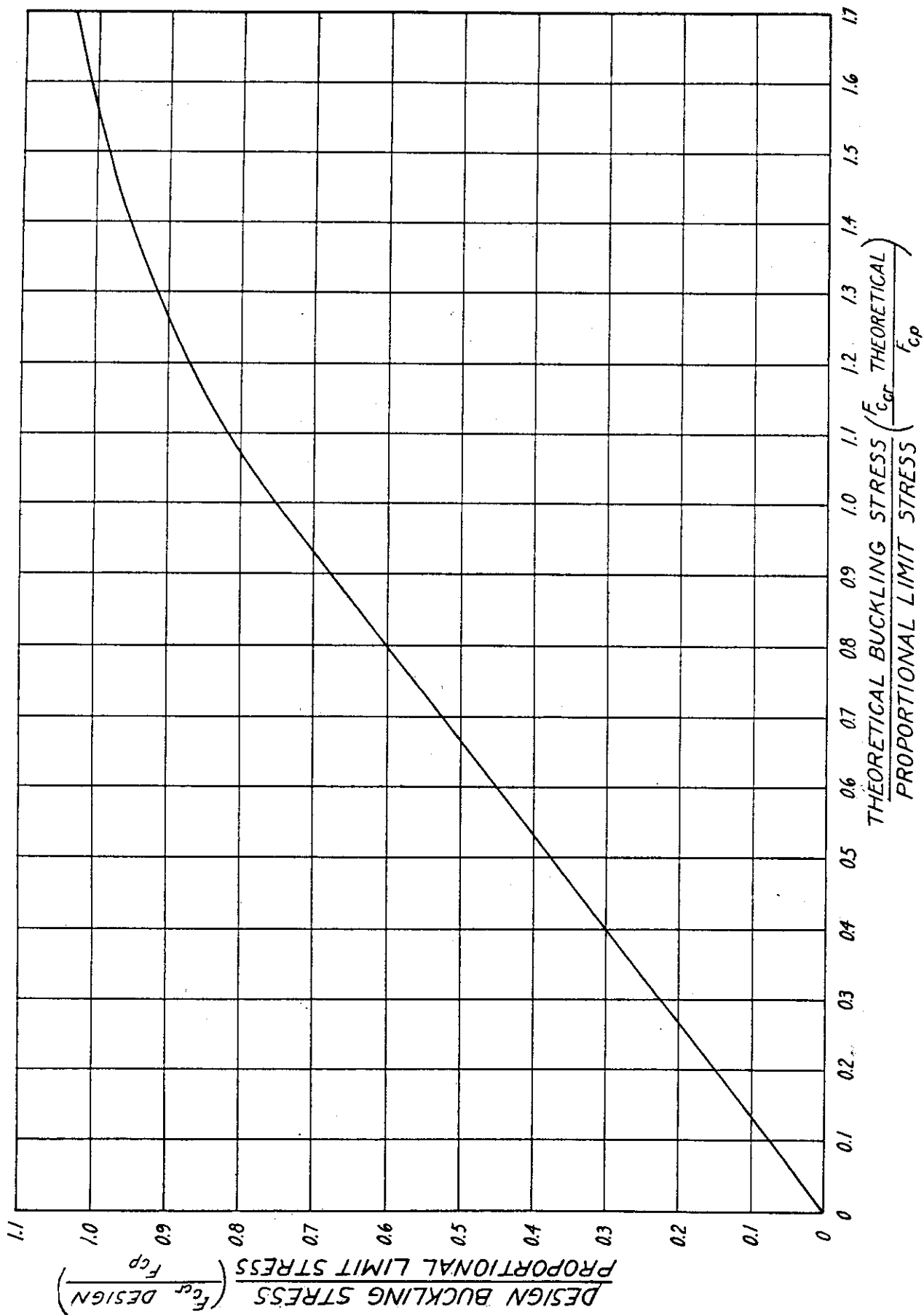


Figure 2-54. Design curve for long, thin-walled plywood cylinders.

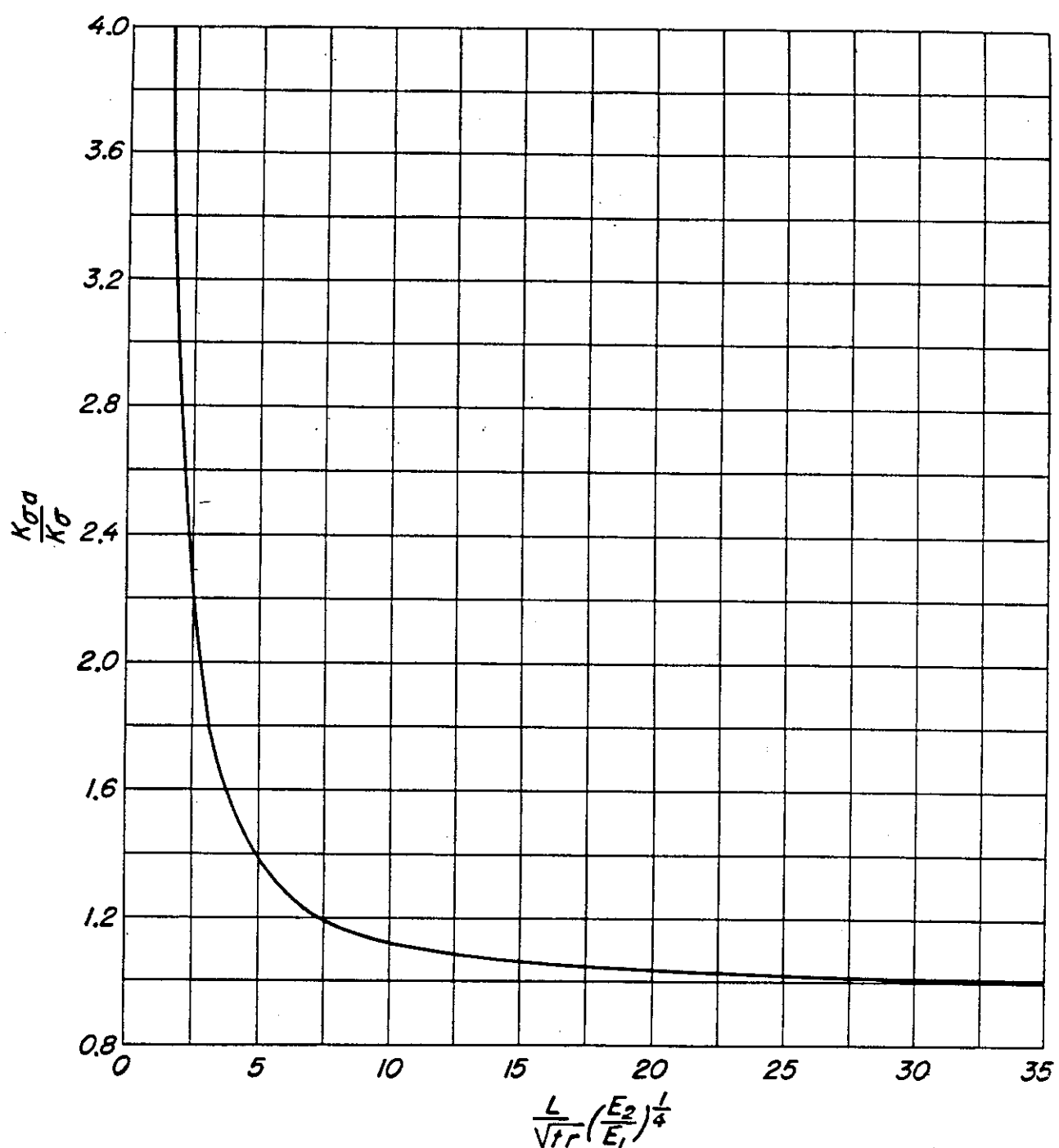


Figure 2-55. Curve showing length effect of cylinders subjected to axial compression.

and that in the stiffeners is

$$f_c = \frac{P E_s}{n h d E_s + 2 \pi r t E_a} \quad (2:117)$$

in which P is the total load on the stiffened cylinder, E_a and E_s apply to the plywood and the stiffeners, respectively, in the direction of the axis of the cylinder, r is the mean radius of the plywood cylinder, n is the number of the stiffeners, h and

d are the cross-sectional dimensions of an individual stiffener, and t is the thickness of the plywood.

2.8412. *Shear stress due to torsion.* The shear stress in the plywood is:

$$f_s = \frac{2 G_{ab} T r_3}{n h G_s \frac{r_2^4 - r_1^4}{r_2 + r_1} + \pi G_{ab} (r_3^4 - r_2^4)} \quad (2:118)$$

and that in the stiffener is:

$$f_s = \frac{2G_s T r_2}{nhG_s \frac{r_2^4 - r_1^4}{r_2 + r_1} + \pi G_{ab}(r_3^4 - r_2^4)} \quad (2:119)$$

in which r_1 is the radius of a cylinder tangent to the inner surfaces of the stiffeners, r_2 and r_3 are the inner and outer radii of the cylinder, respectively, G_{ab} and G_s are the moduli of rigidity of the plywood and the stiffeners, respectively, with reference to longitudinal and circumferential axes, and T is the applied torque.

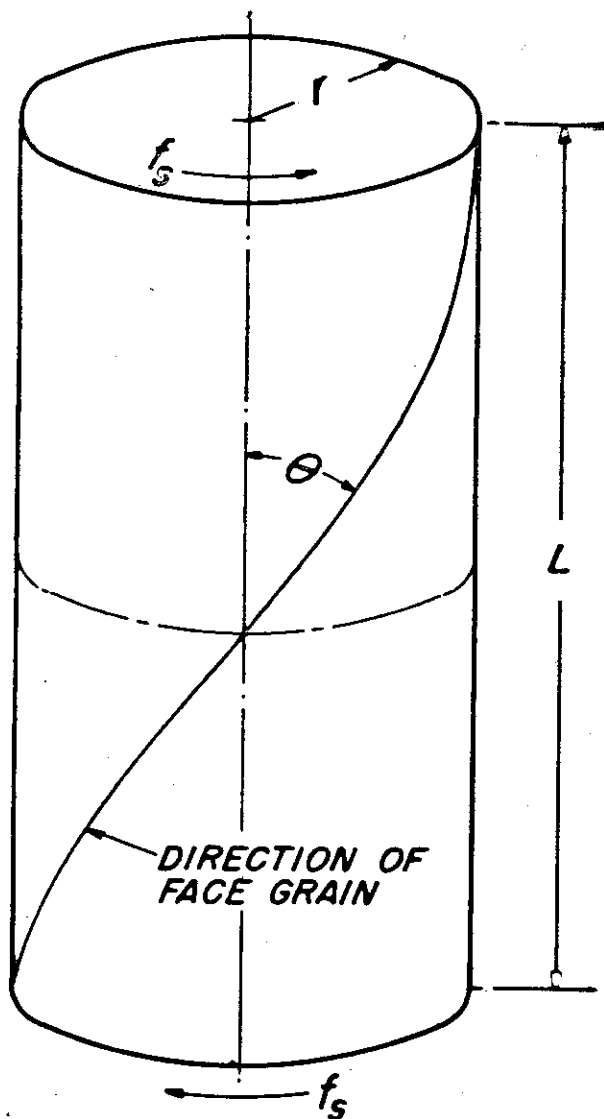


Figure 2-56. Illustration of the meaning of the symbols used for cylinders subjected to torsion.

2.842. *Buckling of stiffened cylinders.*

2.8421. *Axial compression.* In general the plywood will buckle between the stiffeners when the stress in it equals the buckling stress for the

cylinder, the effect of the stiffeners being ignored, and, therefore, formula (2:111) employed. The load at which such buckling occurs can be found by setting this stress (F_{cr} from formula 2:111) equal to f_c in formula (2:116) and solving for the load P .

Tests indicate that unless the stiffeners are quite stiff they will buckle with the cylinder and fail at the load computed in the above manner. If the stiffeners are so stiff that they do not buckle with the cylinder, the maximum load will be greater than that computed. However, no methods are available for the determination of the size stiffeners required to obtain this effect nor to compute the maximum loads that are obtained. (Ref. 2-95)

2.8422. *Torsion.* The shear buckling stress of the curved plywood shell between the stiffeners is about 85 percent of that obtained by formula (2:112) for the cylinder, neglecting the effect of the stiffeners. The torque at which buckling occurs can be found by setting this stress $0.85 \times F_{ser}$ from (2:112) equal to f_s in formula (2:118) and solving for the torque T .

Tests indicate that the maximum torque coincides with the torque at which buckles form.

2.8423. *Bending.* For bending, the design buckling stress determined as for compression may be increased 10 percent. Tests indicate that for very stiff stiffeners this percentage may be increased. (Ref. 2-96)

2.85. STIFFENED CURVED PANELS. A single stiffener bisecting the panel.

2.851. *Axial compression.*

2.8511. *Stiffener axial.* The critical stress in the plywood is computed according to the method of section 2.831, and the critical load of the stiffened panel can be obtained by substituting this value for f_c in equation (2:116), placing n equal to unity, and solving for P .

2.8512. *Stiffener circumferential.* The critical stress is computed according to the method of section 2.831, using the distance between the stiffener and the end of the panel as the length of the cylinder, provided that

$$(EI)_s > 0.4 F_{ccr} r^2 h^2 \quad (2:120)$$

in which (EI) , is given by formula (2.84), and F_{cr} is the critical stress of the entire panel, neglecting the stiffener, computed by the method of section 2.831.

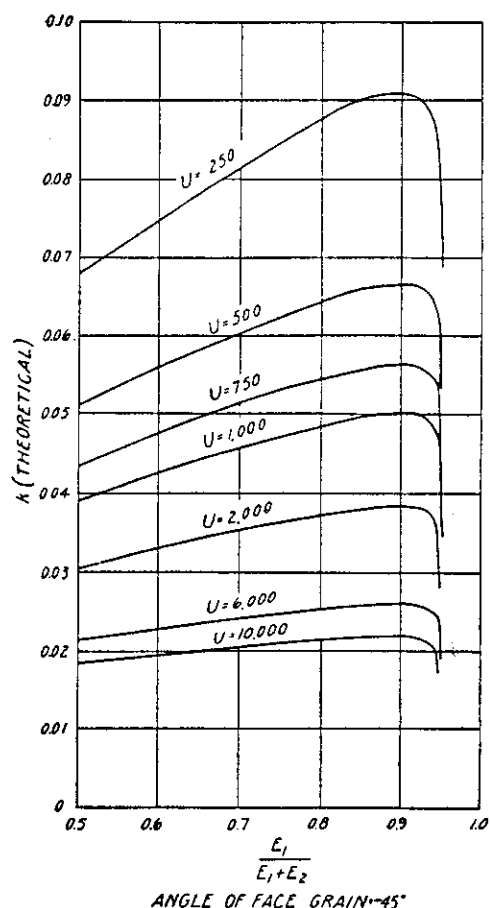
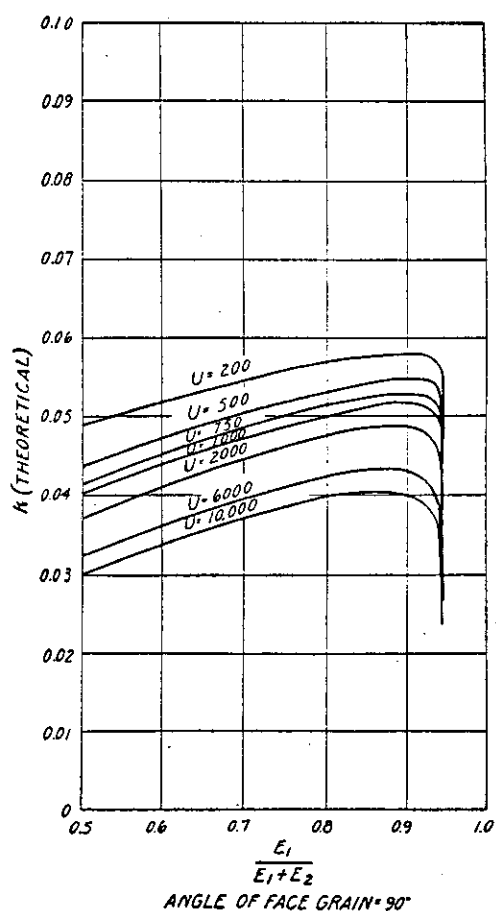
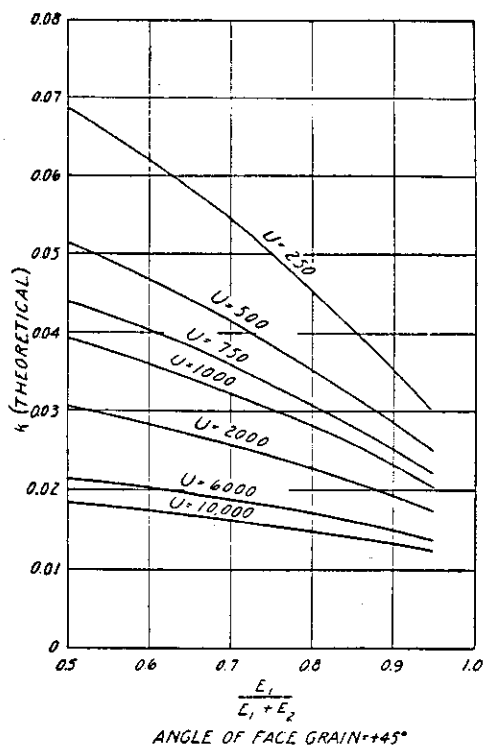
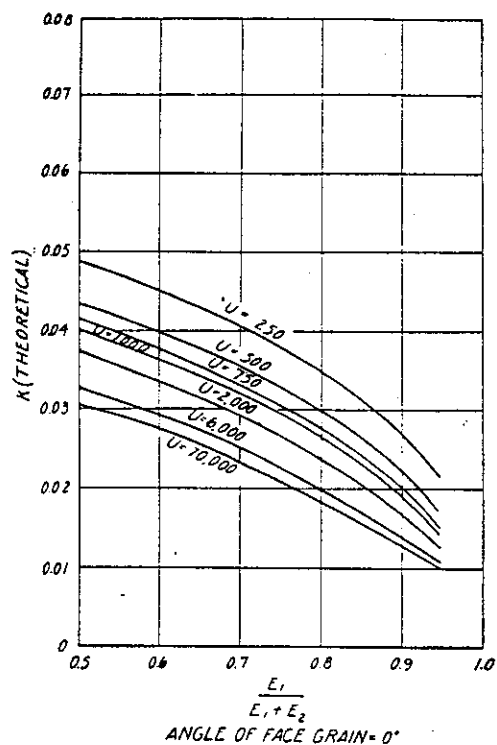


Figure 2-57. Theoretical buckling constants for thin-walled plywood cylinders in torsion, $W=0.036$.